ABSTRACT

Title of dissertation: A BAYESIAN FRAMEWORK FOR ANALYSIS OF PSEUDO-SPATIAL MODELS OF COMPARABLE ENGINEERED SYSTEMS WITH APPLICATION TO SPACECRAFT ANOMALY PREDICTION BASED ON PRECEDENT DATA

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To ensure that estimates of risk and reliability inform design and resource allocation decisions in the development of complex engineering systems, early engagement in the design life cycle is necessary. An unfortunate constraint on the accuracy of such estimates at this stage of concept development is the limited amount of high fidelity design and failure information available on the actual system under development. Applying the human ability to learn from experience and augment our state of knowledge to evolve better solutions mitigates this limitation. However, the challenge lies in formalizing a methodology that takes this highly abstract, but fundamentally human cognitive, ability and extending it to the field of risk analysis while maintaining the tenets of generalization, Bayesian inference, and probabilistic risk analysis.

We introduce an integrated framework for inferring the reliability, or other probabilistic measures of interest, of a new system or a conceptual variant of an existing system. Abstractly, our framework is based on learning from the performance of precedent designs and then applying the acquired knowledge, appropriately adjusted based on degree of relevance, to the inference process.

This dissertation presents a method for inferring properties of the conceptual variant using a pseudo-spatial model that describes the spatial configuration of the family of systems to which the concept belongs. Through non-metric multidimensional scaling, we formulate the pseudo-spatial model based on rank-ordered subjective expert perception of design similarity between systems that elucidate the psychological space of the family. By a novel extension of Kriging methods for analysis of geospatial data to our "pseudo-space of comparable engineered systems", we develop a Bayesian inference model that allows prediction of the probabilistic measure of interest.

A BAYESIAN FRAMEWORK FOR ANALYSIS OF PSEUDO-SPATIAL MODELS OF COMPARABLE ENGINEERED SYSTEMS WITH APPLICATION TO SPACECRAFT ANOMALY PREDICTION BASED ON PRECEDENT DATA

by

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Dedicated to the memory of my father, Dr. Pol Ndu, whose life continues to inspire me...

...and to Alice, my dearest mother, for keeping his spirit alive within us

Acknowlegdements

Reflecting on the experience of researching, developing, and completing this thesis, I have come to the realization that although, a will may indeed provide a way, material, emotional, and divine support are just as consequential in getting to the finish.

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1 Introduction

Learning and adaptation comprise a feedback system through which evolution can be realized. Humans learn from past experiences, adapt to conditions, adverse or otherwise, and ultimately evolve to ensure continued optimality.

This process of evolution through learning and adaptation is reflected in most human endeavors; from military conflict, governance, economic strategy, and financial planning, to architecture, urban development, technology development, and the design of engineering systems. Although far from being a new concept, learning from experience remains an essential and highly utilitarian tool in the set of cognitive abilities humans are endowed with.

Considering the foregoing, it is not surprising that most significant design efforts begin with a precedent analysis geared towards answering the questions: what worked and what failed in the past? Are improvements necessary or is the status quo acceptable? How should improvements, when deemed necessary, be implemented? To address these questions, a comparison between the sources of learning, i.e. the precedent, and the desired optimal design, i.e. the concept, must be carried out. The salient points from the learning and comparison exercise can then be applied to the design effort to evolve an improved or optimal system.

We propose an integrated framework for inferring the reliability, or other probabilistic measures of interest, of a new system or a conceptual variant of an existing system. Abstractly, our framework is based on learning from the performance of precedent designs and then augmenting the limited knowledge regarding the new system with the information available and acquired from the precedent system to enable inference on the properties of the concept. The premise of this thesis is the use of accumulated experiences and engineering knowledge to draw conclusions about the unknown attribute of interest by metering such historical information with a degree of adjustment determined by similarity with the concept. To achieve this, we coalesce elements of generalization theory from the field of psychometrics, classification schemes from phylogeny, spatial data analysis, and uncertainty propagation in subjective inference from the Bayesian paradigm, into a prediction and an analysis framework to formulate and implement this thesis.

1.1 Motivation and Rationale

To ensure that estimates of risk and reliability inform design and resource allocation decisions in the development of complex engineering systems, early engagement in the design life cycle is necessary. An unfortunate constraint on the accuracy of such estimates at this stage of concept development is the limited amount of high fidelity design and failure information available on the actual system under development. Applying the human ability to learn from experience and augment our state of knowledge to evolve better solutions mitigates this limitation. However, the challenge lies in formalizing a methodology that takes this highly abstract, but fundamentally human cognitive, ability and extending it to the field of risk analysis while maintaining the tenets of generalization, Bayesian inference, and probabilistic risk analysis.

Difficulties with transforming the large volume of related but disjointed knowledge pertaining to a system into a structured format for interpretation and use increases the challenge, hence the need for a knowledge collection, organization [1], and knowledge use framework. This thesis is therefore motivated by the need for a formal framework for augmenting limited system information with historical data in order to facilitate an assessment of the chances of success of a system in early development. The rationale being that uncertainty is dictated by degree of knowledge;

- The more we know about a system, the more certain we are about our assessment of its attributes
- Knowledge of a system under development increases as the design itself matures in time.
- The point of least knowledge, and hence highest uncertainty, occurs at the inception of the concept idea
- The value of risk analysis as a decision tool increases as uncertainty increases

The implication of this knowledge-to-uncertainty relationship is that uncertainty is typically elevated at the beginning of large-scale design and development projects, and as a result, large amounts of resources are committed to evaluation of concepts and feasibility studies. We have developed a risk-informed tool for making decisions early in the design process that is applicable at a period of elevated uncertainty, which requires minimal effort but yields higher fidelity results than is presently available,

1.2 Statement of Objectives

This thesis has developed a prediction methodology to support system or product design decisions relatively early in the development process by providing engineers and decision makers estimated success probabilities based on past performance of similar systems. The following sections state the objective of the research.

- Research Objective 1(RO1) - Establish a framework for estimating probabilistic attributes of any conceptual system still in early design, based on a quantified metric of dissimilarity with precedent systems and use of historical performance data. The following are attributes of the prediction framework

  - RO1.1 – quantifies the impact of intrinsic and extrinsic dissimilarity as a measure of difference between system design attributes
  - RO1.2 – allows the introduction of data at any level of a system's hierarchy
  - RO1.3 – is implementable with minimal amount of design or performance data

- Research Objective 2 (RO2) – Establish a formal process for quantifying the level of system dissimilarity using expert opinion and based on a comparison between actual precedent systems and a concept system.
- Research Objective 3 (RO3) - To demonstrate that pseudo-spatial[1] representations, created from subjective expert opinion of proximity of complex engineered systems, provide a mechanism through which the attributes of systems within that pseudo-space can be inferred

1.3 Contributions

This thesis provides the following technical contributions to the field of risk and reliability analysis of products in early design phase:

- A Bayesian reliability analysis framework for complex systems in early design phase that leverages information from multiple precedent systems. The framework is novel in its treatment of the quantification and application of the similarities between the system under development and precedent systems. Our treatment of all parameters in the framework with Bayesian principles results in a probabilistic view of risk that reflects the uncertainties inherent in our estimates.
- A framework that allows use of mixed-level data of many types such that simultaneous updating of probabilistic metrics at all levels of indenture within the system's hierarchy is possible.
- A methodology for converting elicited psychological measures of proximity between engineered systems into quantified distance measures that give rise to the similarity topology of a family of such engineered systems
- An extension and application of spatial modeling and multi-variate analysis in which a recovered pseudo-spatial solution from a multidimensional scaling technique is used to derive Gaussian field representation of a family of complex systems

[1]The phrase *pseudo-spatial* is used to mark the distinction between spatially or geographically referenced data with longitude, latitude, and altitude, and the perceptual map of proximity of entities derived from subjective opinion

1.4 Thesis Organization

This dissertation is organized to emphasis the incremental nature of the process through which the methodology was developed. We open with a traditional introductory section in which the motivation and rationale for pursuing this investigation are laid out. This section also includes a concise set of objectives and contributions.

In Section 2, we discuss a variety of literature that lend, in varying degrees, to the foundational elements of the methodology. This is followed by discussion in Section 3 of the various methods employed in the actual formulation of the method and its implementation.

Sections 4, and 5, provide full description of the methodology including discussion on the various data input and output to be expected from the process.

We provide demonstration and validation exercises in Sections 6, and 7 and close out with a discussion on limitation and possibilities for future work in 8 and 9.

1.5 Notations and Abbreviations

Table 1.1 is a summary of notations and variables that feature in this document. Table 1.2 is a list of abbreviations.

Table 1.1
Table of Notation

Notation	Description
S_i, S_j	i^{th} and j^{th} Stimuli
R_i, R_j	i^{th} and j^{th} Response
D_{ij}	Interpoint distance calculated from from observed dissimilarity
δ_{ij}	subjective observed dissimilarity (expert input)
dij	transformed dissimlairity
N	Number of entities for comparison
Y	Vector of independent random variables of observations
T	Observations at Y
$\hat{T}$	Mean T
$Y(s)$	Observations at s
s	location s
β	vector of regression coefficients
$W(s)$	spatial effects at s
$\epsilon(s)$	error term in stationary Gaussian process at location s
σ^2	Spatial effect variance (partial sill)
$H(\phi)$	covariance function
ϕ	decay rate for detemining range
θ	vector of spatial model parameters
2γ	variogram
$E(x)$	observations at x
r	the distance at which the spatial correlation is negligible (less than .1)
ρ	correlation function parameterized by phi and dependent on di
Σ	Covariance matrix of a multivariate Gaussian
Σ^{-1}	Inverse of the covariance matrix of a multivariate Gaussian
$E[\cdot]$	Expectation
κ	SPDE parameter
τ	non-spatial effect variance, nugget, SPDE parameter
ν	Matérn covariance parameter
τ	non-spatial effect variance, nugget, SPDE parameter
α	SPDE smoothness parameter, related to ν
Δ	the Laplacian in SPDE
Ω	Model performance measure

Table 1.2
Table of Abbreviations

Abbreviations	Description
APFR	Anomaly, Problem, Failure Report
C&DH	Command and Data Handling
Comm.	Communications Subsystem
EPS	Electrical Power System
GF	Gaussian Field
GMRF	Gussian Markov Random Field
GNC	Giudance Navigation and Control
INLA	Integrated Nested Laplace Approximation
MCMC	Markov Chain Monte Carlo
MDS	Metric Multidimensional Scaling
Mech.	Mechanisms and Structures
NMDS	Nonmetric Multidimensional Scaling
PCA	Principal Component Analysis
PO	Polar Ordination
Prop	Propulsion
S/C	Spacecraft
SPDE	Stochastic Partial Differential Equation
TCS	Thermal Control Subsystem

2 Literature Review

2.1 Preamble

The field of engineering is faced with the steady evolution of science and technology. As a result, engineers must deal with the repercussions of adopting new, novel ideas into design solutions. There is, therefore, a need for accurate estimation of the expected behavior of new products owing to evolution from precedents. The ability to estimate a future product's performance metrics is beneficial in making a wide range of decisions; from financial feasibility, risk reduction, and warranty terms, to assessment of design alternatives, simulation of performance, and overall product appeal. To enable such decisions relatively early in product development and prior to commitment of resources, several approaches founded on comparison with existing products have been investigated. Our review of related literature provides a look at different methods developed in the past in support of reliability estimation based on historical information. It also includes a review of literature on classical and Bayesian methods of statistical data modeling and analysis. We additionally present a review of methods for introducing and aggregating expert opinion as a source of data given its importance to the implementation of the proposed methodology,.

Since our research is founded on a comparative assessment of products with a view towards applying historical data to the appropriate elements of a concept product, we seek to develop a method that is based on the degree of similarity between the items being compared. To determine the nature and extent of similarity, and invariably, the degree of adjustment necessary, the proposed methodology explores ideas founded in biological, ecological, behavioral, and psychological sciences, specifically, stimulus-response theories and the concept of generalization of attributes. These ideas integrate concepts from classification, to cladistics, and from gradient analysis to ordination techniques. Appropriately, we present a review of literature of these areas to draw the parallels and extract the extensions to our proposed methodology. Finally, a review of the literature on evaluation of consequence, severity, and criticality of various types of anomalies is presented to provide additional context for the treatment of failure data.

2.2 Risk Analysis Based on Partially Relevant Data

A Bayesian procedure for analyzing failure data of mechanical components in a reliability demonstration test is presented in *Automotive Reliability Inference Based on Past Data and Technical Knowledge* [2]. Making the case that many new products are evolutionary and not revolutionary, Guida and Pulcini postulate that failure data relative to an earlier version of a product, when available, can be used in concert with the designer's confidence in the efficacy of design improvements to judge the reliability of the new version. The procedure allows for inference on the reliability of a new version of an automobile component by using failure data on previous versions and prior information on the effectiveness of design changes that have been introduced in the newer version. The authors first establish a process for making prior inference on the reliability of the new version. By stating the combined impact of prior failure behavior and design modifications on the prediction of future failure behavior, Guida and Pulcini decompose the prior inference in a Bayesian framework into two realms: formulation of a likelihood function which incorporates the past data from a non-homogeneous set of components; and definition of the prior belief on the effectiveness of design modifications.

Of note in the decomposition process is the fact that, in addition to acknowledging the non-homogeneous nature of the component population, the decision is made to model the past data with a time-terminated Bernoulli process with pass/fail criteria determined by component failure before or after the prerequisite time. The latter part of this process limits applicability of the method proposed by the authors to data modeled explicitly via a Bernoulli process.

Usher, Alexander, and Thompson propose a method for predicting system reliability from historical data built on the theory of "competing risk" in *System Reliability Prediction Based on Historical Data*, [3]. Usher et. al describe the development and implementation of a computer-based reliability prediction model designed to utilize historical life-test data to predict reliability of newly developed and untested products at IBM. An aspect of their approach is the added ability for analysis of a "pooled set of life data". The authors define pooled life data as data from different types of systems.

Usher, Alexander, and Thompson present traditional reliability estimation methods and highlight the challenges associated with them in the specific context of early product reliability estimation at IBM. The methods they discussed include life testing to develop characteristics of device life and reliability prediction based on component reliability data and system design and configuration. Given the limitations of the aforementioned methods, Usher, Alexander, and Thompson propose an alternative approach for estimating component reliability through the analysis of historical system life data.

Usher, Alexander, and Thompson propose an early phase reliability prediction based on a scalable hierarchy that maps the system-to-component hierarchy of the concept. The authors list the implementation issues associated the proposed model. The first is the classification of large numbers of components into categories and assuming that the components in each category have identical life distributions. Although this reduces the number of parameters that would have to be estimated for the components, it begs the question of the applicability of the model. The second implementation issue is identifying instances of failure for every component or component category. This issue leads to the concept of pooling data. Component failure data from different systems tests are pooled together to get a more comprehensive set of failures that would include all the component categories. The final implementation issue is the masking of specific component failures when the failure root cause is not identifiable. Usher, Alexander, and Thompson point to literature where general likelihood expressions for masked data have been explored in the case of a series-system of three components with exponentially distributed lives. The results of the work lead Usher, Alexander, and Thompson to conclude that maximum likelihood analysis of masked data will require complex numerical procedures. As a consequence, they state as a necessity the need to find the exact cause of a system's failure and ascribe it to the right component.

Miyakawa presents parametric and nonparametric methods for reliability estimation in a competing risk scenario and with incomplete data in *Analysis of Incomplete Data in Competing Risks Model* [4]. Specifically, consideration is given to cases where failure times are observed but not the actual failure cause. Maximum likelihood estimators are developed, in the case of a two-failure mode system, for the failure rates of components within the system. We apply Bayesian methodology rather than Miyakawa's maximum likelihood estimators to this competing risk approach for treatment of masked data such as historical observations at a system level.

Neil et. al.'s *Using Bayesian Belief Networks to Predict the Reliability of Military Vehicles* [5] presents the use of a Bayesian Belief Network (BBN) as a means of incorporating all available and relevant evidence into the reliability and maintainability assessment of proposed United Kingdom Ministry of Defense military vehicles. The proposed approach seeks to combine "hard" information (failure counts, modes, and exposure periods), used in traditional reliability analysis, with "soft" information (manufacturer reputation, design staff experience, etc.). The approach rides on the fact that Bayesian probability allows the expression of uncertainty with a unifying framework. The result of the method development effort is a software tool, Transport Reliability Assessment and Calculation System (TRACS) that predicts the probability that non-combat land vehicles will meet their mission requirements using soft and hard data in a single decision model. The BBN approach proposed by Neil et. al. for estimating the parameter of interest, a failure rate in this instance, is analogous to a hierarchical Bayesian modeling. The intent in the Neil approach is "learning a failure rate distribution from samples of similar subsystems"[5]. In the TRACS BBN, weights are used to model bias towards data sources based on the subjective belief on the degree of applicability to the unknown system. In hierarchical Bayesian modeling this is similar to making inference on a specific individual's trait based on traits exhibited by a group to which the individual belongs. Neil et al, however do not provide an approach for eliciting the subjective opinion and transforming it to the bias defining weights.

Lough et. al. provide a study on the relationship between function and risk in early design in *The Risk in Early Design Method*, [6]. The authors present a mathematical construct for mapping product function to risk assessment, which can be used in the conceptual design phase. The method is aimed at enabling a preliminary risk assessment that can be used to, not only identify risks, but also to reduce the subjectivity of the likelihood and consequence value of a risk. The Risk in Early Design (RED) utilizes the 5x5 risk grid approach introduced by the 'Risk Management for Defense Acquisition' (Office of the Under Secretary of Defense 1999) in which risk is presented as a product of likelihood, consequence, and severity. The RED method provides closed form mathematical equations for estimating the so-called L1-Prod, L2-Agg, C1-Max, and C2-Agg which respectively refer to the first and second likelihood mapping and the first and second consequence mapping from historical systems to the product under development.

The concept of functional mapping as a means of comparing existing systems and less mature design concepts has veritable importance to our proposed methodology since the high level functional requirements of any concept can be defined even in the absence of specific design detail. The issue with the risk estimation method proposed by Lough, et. al. is that it yields point estimates that convey neither the aleatory nor epistemic uncertainties attendant in the method's representation of the system's failure processes and the state-of-knowledge regarding the risk elements. By applying Bayesian probabilistic methods, our proposed method allows for the inclusion and propagation of uncertainty in the estimation of risk elements.

The Groen et. al. report *Reliability Data Collection and Analysis System* [7] describes the Reliability Data Collection and Analysis System (ReDCAS) software tool developed for Ford Motor Company for collection and analysis of reliability data. The tool leverages Bayesian data analysis methods to predict reliability based on warranty data, test data, and engineering judgment. ReDCAS has been used for performing reliability assessments for products in development. ReDCAS is structured to enable assessment of the reliability of components that are in the early stage of design despite lack of data from the component itself. Developers of the ReDCAS methodology posit that if a reliability assessment for future products is desired, the reliability behavior observed for existing products can provide a source of evidence as long as perceived differences are accounted for. A relevance factor, akin to the weights used to account for bias in [5], is used to describe the applicability of the data emanating from historical comparators and scale the impact of the data. The same issue of eliciting and quantifying the subjective opinion on the relevance factor present in the [5] is also attendant in [7].

Pan and Sanchez proposed a method in their paper titled *An Enhanced Parenting Process: Predicting Reliability in Products Design Phase*[8] in which, again, the resounding issues associated with reliability prediction at a product's early design stage are acknowledged and referenced as motivation. The authors propose an approach to predicting a new product's reliability in early development by using reliability information from the existing products, referred to as "parents", in the so-called "parenting process".

Pan and Sanchez integrate the mathematical foundation of the parenting process with an expert opinion elicitation method to formulate a strategy in which a new product with similar reliability or failure structure as its parent product is examined. The reliability or failure structure is used to determine the relationship between failure modes (m_i) and causes (c_i). Expert opinion is then used to evaluate the impact of design changes and improvements on each failure cause by comparing parent and child, and finally the reliability of the new product is estimated.

Parent selection defines a baseline reliability structure of the new product. On the premise that if no new failure modes are introduced due to design changes, the reliability structure of the new product is definitive and sets the basis for reliability estimation.

Pan and Sanchez propose a process for eliciting expert opinion based on the guidelines and principles put forth by [9]. The survey elicits two responses from each expert, a "best estimate" for the median of the parameter that represents the magnitude of change from the parent to the new product for the failure cause, and a "degree of uncertainty" associated with the estimate. These two values are treated as the parameters of the distribution of a "parent factor", which is then used as a multiplier for scaling the parameter of interest for the new product.

In reviewing relevant works in early design phase reliability analysis, numerous approaches of reliability prediction based on historical data in other industries have been proposed. Guida and Pulcini proposed a method relying on Bayesian inference, which provides an approach for quantifying and propagating of uncertainty within a Bayesian framework. Usher, Alexander, and Thompson, provide an approach that relies on the analyst's ability to trace and ascribe exact failure causes to culprit systems and then using maximum likelihood estimation procedures, identify the parameters of interest on which to build the necessary predictive model.

Usher et. al., Miyakawa et. al. discuss methods of addressing masked data from life test, while Neil et. al. also cast the use of historical data in a Bayesian framework. Lough et. al describe a concept product in terms of its intended functions but quantify risk through a formalism based on the “risk index” and “likelihood and severity” paradigm. Groen and Droguett’s ReDCAS software tool extend the boundaries of use of historical data similar to [5] by adopting Bayesian inference combined a weighted posterior method for aggregating data and expert opinion. A benefit of the ReDCAS methodology is the analyst’s ability to make use of the extensive warranty information on generations of a heritage product with chronologically decreasing dissimilarity. The large amount of data available for the implementation of the ReDCAS allows the development of failure rate ratio of successive generations as a measure reliability impact [7]. This however confines the method to analysis in a data-rich environment. ReDCAS also allows the assessment of the impact of design fixes and results of extensive test programs. Despite the promising attributes of ReDCAS, the question of how a method reliant on historical-data for reliability assessment can be implemented in a data-poor or data-rich and database agnostic environment remains.

Although not explicitly presented as a method of reliability estimation, Mosleh and Droguett, in *Bayesian Treatment of Model Uncertainty for Partially Applicable Models*[10] extend their initial work in [11] to incorporate additional types of information about a model such as subjective views pertaining to model credibility and applicability outside the domain of its intended use. This extension provides a comparative view of models where the possibility of estimating an unknown of interest exists from various models. The parallels from this construct to the proposed research are easily drawn; using subjective knowledge and other available data regarding the relatedness of two or more well-defined systems, models, or attributes, to infer the nature of a similar but less well-defined system, model, or attribute.

2.3 Theories of Learning, Stimulus-Response, and Generalization

In *Towards a Statistical Theory of Learning* [12], Estes proposes a form for all fundamental laws that relate behavioral response, R, and environmental stimulus, S, variables; where response behavior is a function of environmental stimulus. He maintains that all response-inferred laws must be based on such a relationship but points out the issues that attend the simplified view of stimulus and response as reducible units. Issues such as the need to hypothesize the processes that account for variations in observed responses or behavior. In offering an approach to address this issue, Estes adopts a statistical interpretation of stimulus-response, an interpretation that by its stochastic nature accounts for the variability in response and stimulus, to derive quantitative laws, which dictate behavior systems.

In this formalism, records of behavior are regarded as dependent variables possessing quantitative properties, while statistical distributions of environmental events are independent variables. Through this construct, the probability relations between changes in behavioral and environmental variables are obtainable, specifically, the probability of a response. Estes however provided no rationale for the internal workings of organisms; rather he proposed "that the theory be evaluated solely by its fruitfulness in generating quantitative functions relating various phenomena of learning and discrimination" [12].

Roger Shepard in *Stimulus and Response Generalization: A Stochastic Model Relating Generalization to Distance in Psychological Space* [13] introduces the concept of "psychological distances" as an alternative approach to applying stochastic models of learning, as proposed in [12], to generalization phenomena. According to Shepard, in a stimulus-response process, $(S - R)$, the error in which the response assigned to, or expected of, a stimulus S_j, follows the presentation of another, S_i, is known as generalization errors and the probability of generalization errors decrease with decreasing dissimilarity between the two stimuli. In lieu of dissimilarities, Shepard introduces the concept of distances between stimuli. These distance measures most conform to explicit metric axioms in [13]. Shepard states, "Any set of elements for which a distance function that satisfies the metric axioms has been defined is called a metric space. The space may be called a physical or psychological space depending upon whether the distances are determined from physical or psychological data." Judgments of psychological distance, i.e. psychological data, are obtained in terms of dis/similarity [14]. Shepard assumes that there is a function that relates the conditional probability of generalization error (that a response, R_j, will be elicited from a stimulus, S_i that has its own assigned response, R_i) to the inter-stimuli distance, $D_{i,j}$.

Figure **??**, recreated from [13], illustrates the $S - R$ process in which every stimulus has an associated response. It follows from Shepards Theory of Generalization that response confusability can be dictated by stimulus confusability and that the degree of confusability increases or decreases as the similarity of stimuli increases.

Our objective is the inference of the unknown response R_j, given the known response, R_i, that has been associated with S_i, and the measure of proximity between the stimuli, D_{ij}. R_j is a monotonicaly decreasing function of D_{ij} and R_i and we must determine the pseudo-spatial arrangement of designs or stimuli that will enable quantification of D_{ij}, and subsequently determine the function that maps R_i to R_j.

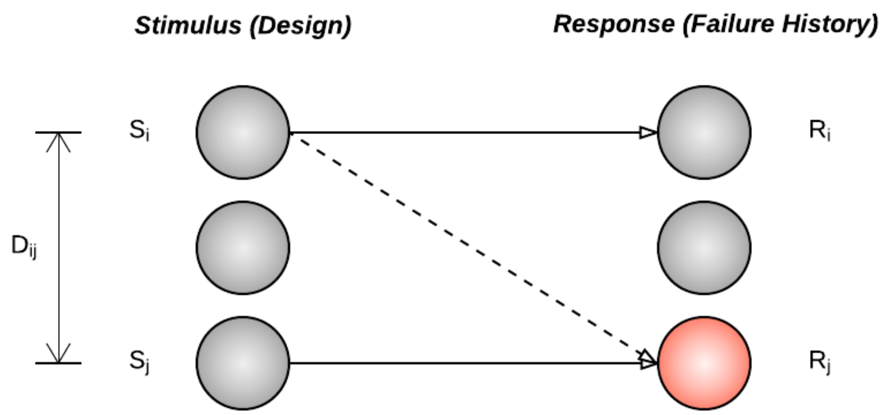


Figure 2.1

Stimulus-Response confusion as a function of proximity. Source: [Shepard, 1957]

Though previous studies indicate that this function is a monotonically decreasing function, Shepard points out the lack of consistency in specifying its form with precision. He attributes this to the fact that most measures of dissimilarity are derived from physical scale data and the number of noticeable differences. Alternatively, Shepard adopts a process of estimating the so-called psychological distance between stimuli and then progresses to using multidimensional scaling methods to convert psychological data or similarity judgments to inter-stimulus distances [13], [14]. By starting with the probability of generalization errors, Shepard proceeds in reverse to determine the function, which will transform the probabilities into distance measures that satisfy the metric axioms. With additional assumptions introduced to increase the stringency of the metric axioms, he posits that an exponential decay function describes the generalization relationship. Shepard points to data from other generalization studies that are consistent with this premise.

Toward A Universal Law of Generalization for Psychological Science [15] is a treatise, supporting the proposed universal law, in which a psychological space is resolved for any set of stimuli based on metric measures of separation between the stimuli "...such that the probability that a response learned to any stimulus will generalize to any other is a monotonically decreasing function of the distance between the pair of stimuli".

Positing on the primacy of generalization, Shepard's states, "Differences in the way individuals of different species represent the same physical situation implicate, in each individual, an internal metric of similarity between possible situations". Researchers in psychology "have obtained empirical *gradients of stimulus generalization* relating the strength, probability, or speed of a learned response to some measure of difference between each test stimulus and the original training stimulus".

Shepard introduces his premise by first obviating the conclusions of behavioral scientists, Karl S. Lashley, and the mathematical learning theorists, Robert R. Bush and Fredrick Mosteller, regarding the noninvariance of generalization as a concept. Their conclusions where based on research results defining the independent variable in a generalization gradient as the physical differences between stimuli and these results revealed wildly varying generalization gradients, some even nonmonotonic in any direction.

As an alternative to Lashley, Bush, and Mosteller, Shepard proposes that for a law to be invariant across perceptual dimensions, or other entities, it must be formulated "with respect to the appropriate abstract psychological space". He attributes the variations in previously attained gradients of generalization to differences in the psychophysical function, which operates uniquely for individuals, in mapping physical parameters to psychological space. On the assumption that if a purely psychological function relates generalization to distance in a psychological space, then invariance of the law would be achieved.

Shepard approached the proof of the law's universality by considering generalization data as his starting point and then investigating if there is a monotonic function whose inverse will transform the data into distances in a metric space. By applying his, and Kruskal's multi-dimensional scaling ordination techniques, Shepard uncovers the universality of the exponential law relating gradients of generalization to psychological space. Figure 2.2 are Shepard's plots of generalization gradient data demonstrating the monotonic exponential decay behavior.

The remainder of the paper works out the mathematical formalism that underpins Shepard's derivation of the exponential law. He defines a consequential region as the space around an entity's psychological space around which generalization can be made on the basis that the "psychophysical function that maps physical parameter space into a species' psychological space has been shaped over evolutionary history so that consequential regions for that species, although variously shaped, are not consistently elongated or flattened in particular directions". Shepard dictates the following conditions regarding this region of consequence:

1. All locations are equally probable
2. The probability that the region has a size, s, is given by the density function, $p(s)$ with a finite expectation μ
3. The region is convex, finite, and centrally symmetric

Owing to condition 2, the conditional probability that x is contained in the consequential

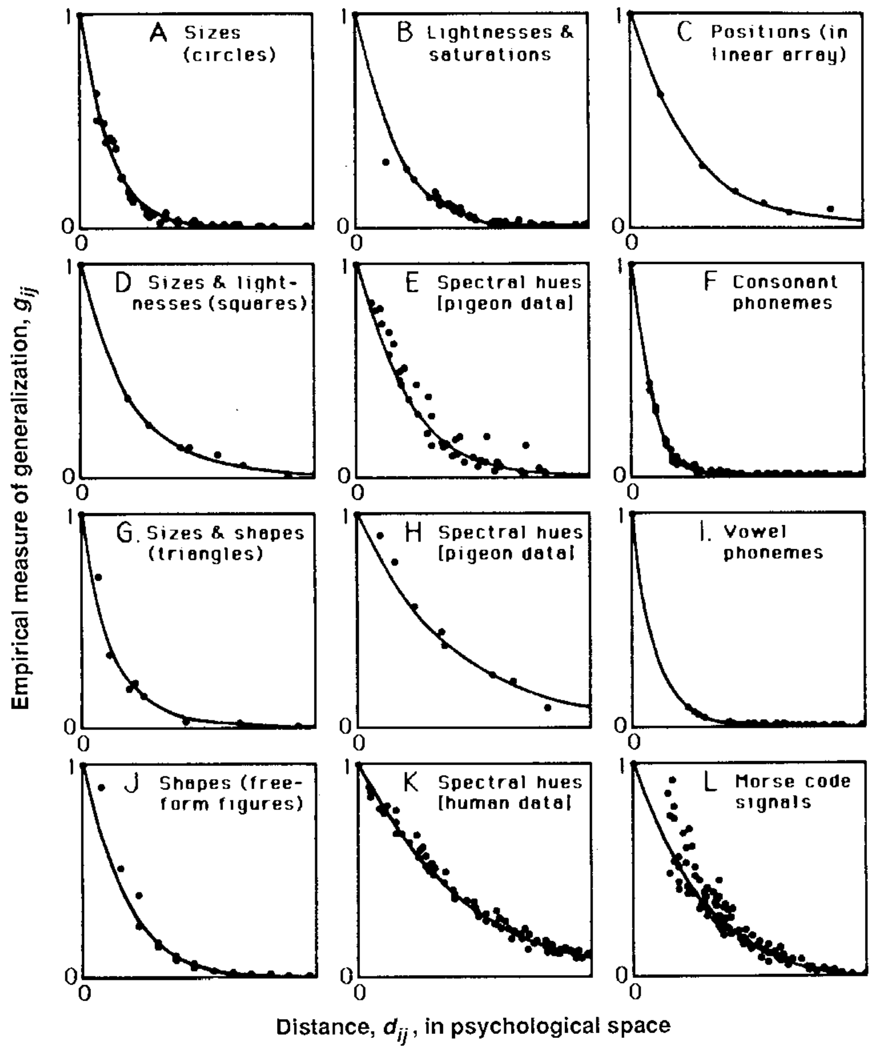


Figure 2.2

Generalization gradients (Source: [@Shepard1987])

region is the measure of the overlap to the whole region [15] $\frac{m(s,x)}{m(x)}$, see Figure 2.3 (recreated from [15]).

By integrating over all possibilities of s given prior belief about $p(s)$, Shepard concludes that the result is the probability that a response learned to the stimulus, 0 (for example, a precedent characteristic centered at 0), will generalize to x, $g(x) = \int\limits_0^\infty p(s) \frac{m(s,x)}{m(x)} ds$ (the concept's characteristic at a distance x from the precedent's characteristic centered at 0). This expression for the probability of generalization from stimulus 0 to x satisfies the law of total probability:

$$\int_0^\infty p(s)ds = 1$$

and,

$$\int_0^\infty s \cdot p(s)ds = \mu < \infty$$

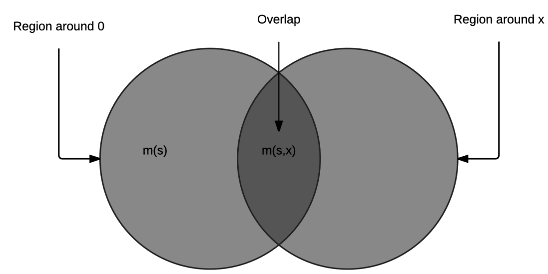


Figure 2.3

The volumetric measure of ovelap indicates the conditional probability of interest (Source: [@Shepard1987])

Shepard continues for the one-dimensional case, stating that the consequential region is an interval of length, s, and the measure of overlap is $s - |x|$ such that $g(x)$ becomes:

$$g(x) = \int_0^\infty p(s)\frac{s - |x|}{s}ds$$

For a one-dimensional space, the distance between the two stimuli, 0 and x, as derived by Shepard is $d = |x|$, and thus:

$$g(d) = \int_0^\infty p(s)ds - d\int_0^\infty \frac{p(s)}{s}ds$$

The underlying probability density function in the expression for $g(d)$, $p(s)$ determines the generalization function, however, Shepard found that an exponential decay function reasonably describes $g(d)$ regardless of the choice of $p(s)$, however the Erlang probability density function (with a shape parameter of 2) exactly yields an exponential form for $g(d)$:

$$p(s) = (\frac{2}{\mu})^2 s \times e^{-\frac{2}{\mu}s}$$

and

$$g(d) = e^{-2\frac{d}{\mu}s}$$

In this thesis, we apply the notion of a universal generalization gradient as a function of the quantified similarity or distance between two stimuli to the present problem of inferring the behavior of a conceptual engineered system based on the behavior of precedent systems. This extends the concept of generalization from sentient or living organisms to non-biological systems by leveraging human cognition and the ability to affect system design as the bridge between Shepard's application and ours.

2.4 Ordination Techniques

Ordination is the arrangement or *ordering* of species or sample units along gradients [16] with a view to representing typically high-dimensional sample and species relationships in much lower-dimensional space. We review relevant literature on ordination which inform our approach to reconstructing the Euclidean spatial configuration of entities under comparison. Although used largely as reduction techniques for high-dimension data, our interest in these methods stems from:

1. The possibility of uncovering metric measures of similarity from psychological perceptions of nearness
2. Determining the underlying parameter in the relationship between these metric measures and the probability of generalization for each case of comparison

With our focus on quantification of similarities in relation to distances between entities, we proceed by examining distance-based ordination techniques. Methods such as Principal Coordinate Analysis (PCA), Polar Ordination (PO), Metric Multidimensional Scaling, and Nonmetric Multidimensional Scaling (NMDS) are all commonly used approaches for reducing the dimensionality of data, however the ability to achieve this reduction by rank ordering intuitive measures of proximity makes NMDS most appropriate for our research due to the rank-ordered nature of the expert opinion used in the methodology.

Our review begins with an in-depth look at the seminal works in the literature that propelled NMDS. In *The Analysis of Proximtities: Multidimensional Scaling with an Unknown Scaling Function. I* [17], Shepard describes a process, which he calls *Analysis of Proximtities*, intended to "reconstruct the metric configuration of a set of points on the basis of nonmetric information about that configuration". His proposed program is intended for analysis of psychological data which reflects the degree of similarity between stimuli, thereby revealing the underlying structure of such data. Shepard adopts the more abstract and generic term *proximity* to be inclusive of both measures of association and of similarity. Accordingly, proximity captures the idea of psychological nearness, closeness, or degree of proximity, [17], and Shepard calls the number "representing the closeness of the relation between a pair of entities" a *proximity measure* for that pair.

NMDS methods move n points representing the n stimuli (usually by steepest descent) until the stationary configuration is achieved that minimizes an explicitly defined measure of departure from a monotonic relation between the generalization measures g_{ij} and the corresponding distances d_{ij}, [15].

The plot of the generalization measures g_{ij} against the distances d_{ij} between points in the resulting configuration is interpreted as the gradient of generalization. It is a psychological rather than a psychophysical function because it can be determined in the absence of any physical measurements on the stimuli [15].

First acceding that although an objective definition of proximity is largely applicable to objects in physical space, the natural tendency is to extend this notion to situations where the physical representation of a measure of nearness is not explicit. Shepard postulates that this natural tendency is a result of "a rough isomorphism between the constraints that seem to govern all of these measures of similarity or association, on the one hand, and the metric axioms (which formalize some of the most fundamental properties of physical space), on the other". He then offers the following thoughts to illustrate the loss of precision that gives rise to this isomorphism: "to the metric requirement that distance be symmetric, there is the corresponding intuition that if A is near B then B is also near A. To the metric requirement that the length of one side of a triangle cannot exceed the sum of the other two, there is the corresponding intuition that, if A is close to B and B to C, then A must be at least, moderately close to C".

According to Shepard, the use of the words "very" and "moderately", points to the shift from the objectively defined concept of distance to the more psychological concept of proximity. However he is motivated to apply the well-established quantitative methods for assessing metric distance into the more nebulous area of psychological perceptions of proximity. This motivation is driven by the need for a data reduction approach in analysis of proximity data. Shepard cites research in the investigation of factors that dictate confusion between Morse code signals where there are 36 individual signals but an immense 630 pairwise similarity measures between them. This turns the investigation of patterns or structure into a rather onerous effort. He concludes that if an underlying spatial structure can be discovered, then the path would be opened for investigation of the structure's relationship with the physical properties of the stimuli. Shepard's method seeks to find the "monotonic transformation of the proximity measures" through a **distance function** that would convert the psychological data to explicit distance measures such that the spatial structure contained latently in the psychological data is recoverable. This would result in a reduction of the data since the initial large number of proximity measures can be reconstructed from a smaller set of coordinate points for the Euclidean space. Shepard lists his three paramount objectives:

1. Minimum dimensionality in Euclidean space such that the distances are monotonically related to the proximity measures
2. A set of coordinates for points in this space
3. A plot showing the shape of the initially unknown function relating proximity to distance

The remainder of [17] is devoted to the mathematical formalism which underscores the *Analysis of Proximities* method such that it achieves the intended objectives. In the sequel paper to [17], *The Analysis of Proximtities: Multidimensional Scaling with an Unknown Scaling Function. II* [18], Shepard demonstrated applications of the methodology in two cases. By first applying it to data simulated from monotonically transforming the interpoint distances in a known spatial configuration, he shows that recovering the original metric configuration is independent of the distance function used to transform the data. The second, and more relevant to our goals, is the application to measures of inter-stimulus similarity and confusability (probability of generalization errors) obtained from psychological experiments.

Multidimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis by J. B. Kruskal [19] is another seminal work on ordination methods in search of a method for representing a set of objects geometrically by a set of points equal to the number of objects such that the interpoint distances are indicative of the similarities they share. Kruskal sets out to establish that a monotone relationship, increasing or decreasing, exists between measurements of similarity, dissimilarity, confusion probabilities, correlation coefficients, or dissociations and the distances in the spatial configuration of the interpoint distances.

He points to the advances made by Shepard in [17] towards the goal of establishing monotonicity between similarity and distance. These advances resulted in demonstration of the fact that rank order of similarities or dissimilarities is sufficient to determining a satisfactory spatial configuration. Kruskal contends that Shepard offered no mathematically definitive intimation of what constitutes a solution. By focusing on Shepard's concept of a measure of departure from the condition of montonicity, Kruskal arrives at a technique that minimizes this measure of departure through the use of least-squares regression. Essentially, Kruskal's method incorporates performing a "monotone regression of distance upon dissimilarity and use if the residual variance,. . . , as our quantitative measure". Kruskal terms this element the **stress**, which is a measure of how well the proposed spatial configuration matches the initial proximity data. On defining a minimum acceptable *stress*, the solution is regarded as the best fitting configuration of points. We utilize Kruskal's measure of stress in evaluating the adequacy of the resulting spatial configuration.

2.5 Uncertainty Analysis

Droguett and Mosleh present a Bayesian methodology for the assessment of model uncertainty where models are treated as sources of information on the unknown of interest in *Bayesian Methodology for Model Uncertainty Using Model Performance Data*, [11]. This framework is applied to a case where models provide point estimates about an unknown and information about model performance are available in the form of pairs of experimental observations and model predictions [6]. We extend the approach proposed by Droguett and Mosleh in evaluating the associated uncertainty in the performance of our methodology.

US Nuclear Regulatory Commission Regulation (NUREG), NUREG-1855 [20], *Guidance on the Treatment of Uncertainties Associated with PRAs in Risk-Informed Decision Making* main report authored by Drouin et. al. provides relevant guidance on the modeling and identification of the different sources of epistemic uncertainty; parameter, model, and completeness. It also provides different approaches for addressing them. For example, in characterizing the parameter uncertainty associated with a PRA basic event, uncertainty is introduced via the choice of the basic event model and via the choice of the parameters within the model. Three methods for describing the uncertainty of parameters within basic event models are proposed in NUREG-1855; 1) the frequentist method, 2) Bayesian updating, and 3) expert judgment. We adapt the latter two in the development of our framework.

Smith presents an approach for characterizing the uncertainties in an analytic model by using a multivariate Taylor series expansion implemented through a spreadsheet package *Uncertainty Propagation Using Taylor Series Expansion and a Spreadsheet* [21]. The fundamentals of the method are easily transferable to modern spreadsheet packages and other scientific analysis tools.

Smith's approach is based on the premise that if a representative mathematical formula exists for a system, or an attribute of a system, then a value for that system or system attribute can be obtained by evaluating the formula using estimates for the variables in the formula. Recognizing the widely held belief in the risk analysis community that point estimates lack credibility without justification for their selection over other possibilities, Smith proffers the Taylor series expansion method via spreadsheet implementation as means to addressing the uncertainty with using point estimates. Smith's approach harkens to simulation methods that form the bedrock of Bayesian computation and the evaluation of non-closed form multivariable integrands.

Recognizing that decisions are sometimes based on beliefs concerning the likelihood of uncertain events Tversky et. al investigate the determinant of such beliefs in their paper, *Judgment under Uncertainty: Heuristics and Biases*, [22]. In asserting that subjective assessment of probability is similar to the subjective assessment of physical quantities, such as distance, Tversky et. al. claim that judgments are based on data of limited validity, which are governed by heuristic rules. The authors describe three heuristics that are relevant in the assessment of probabilities; representativeness, availability, and adjustment or anchoring. We apply these heuristics in the evaluation of expert opinion.

These papers and articles offer approaches for addressing the various types of uncertainty that can be anticipated in modeling, analysis, and expert judgment. From model uncertainty to error propagation through parameters, we are presented with methods for guarding against misleading results from data use.

2.6 Anomaly Effect and Criticality

Haga and Saleh apply the concepts of epidemiology – the study of the patterns, causes, and effects, of health and disease conditions in defined populations – to a population of geosynchronous communications spacecraft and its on-orbit anomalies and failures in *Epidemiology of Satellite Anomalies and Failures: A Subsystem-centric Approach*, [23]. This work provides insight to the prevalence of different types of anomalies across spacecraft subsystems. Lutz et. al. present the results of an investigation of safety-critical software anomalies occurring during operations in the similarly titled *Empirical Analysis of Safety-Critical Anomalies During Operation* [24]. Drawing data from Jet Propulsion Laboratory's (JPL) institutional database of anomaly reports for multiple missions, the authors base their study on existing literature on defect analysis methods, specifically, the Orthogonal Defect Classification (ODC) developed at IBM. The ODC method provides a means of "extracting signatures from defects" and to correlate the defects to attributes of the development process.

A 2005 study of on-orbit spacecraft failures resulted in the identification of 156 failures from 1980 to 2005 on civil and military satellites [25]. Tafazoli analyzes these failures to compare different spacecraft subsystems and estimate their impact on the mission in *A Study of On-Orbit Spacecraft Failures*. Grottke et. al. analyze faults discovered in the on-board software for 18 JPL missions. These faults were documented in over 13,000 anomaly reports recorded after launch and operationalization of the systems. Grottke et al. present the proportion of different types of faults and their time-dependent evolution. They also provide definitions of three distinct types of software faults; Bohrbugs, Mandelbugs, and Aging-related bugs in *An Empirical Investigation of Fault Types in Space Mission System Software*, [24].

A fundamental piece of the framework proposed in [24] is the analysis and assessment of the impact of documented anomalies and failures with the objective of ascribing to specific parts of a concept product. The works reviewed in this section provide a reference base for quantifying the criticality of these observed anomalies.

2.7 Elicitation and Use of Expert Opinion as Evidence in a Bayesian Framework

Elicitation and use of expert opinion can be divided into two broad categories; mathematical and behavioral approaches. While mathematical methods individually elicit opinion on probabilities and then apply mathematics to combine and aggregate subjective assessments, behavioral methods seek to build consensus of opinion. Among mathematical approaches are Bayesian methods and non-Bayesian axiomatic methods. Behavioral methods include the Delphi method and the Nominal Group method. Fumika Ouchi presented a literature review on the use of expert opinion in probabilistic risk analysis in a World Bank Policy Research working paper, [26], in which several important works on the topic were addressed extensively.

Mathematical methods for aggregating and incorporating expert opinion were presented by Mosleh and Apostolakis in [27]. Mosleh and Apostolakis proposed a model for the use of expert opinion in their paper. The authors propose a Bayesian framework in which expert estimates are treated as evidence that must be evaluated by a decision-maker and incorporated into existing body of knowledge. The Bayesian paradigm presented a natural process for implementing this model and they subsequently proposed approaches based on the normal and log-normal likelihood functions.

In *Expert Elicitation for Reliable System Design*, [28] Bedford et al review the role of expert judgement in support of reliability assessments within the systems engineering design process. They differentiate between the role of expert judgment in the design context versus in risk assessment by considering the former to be more like statistical process control than pure statistical assessment of an unknown.

Mosleh and Apostolakis, [29], applied a method for assessment of probability distributions derived from expert opinion to the assessment of seismic fragility curves. Their methodology hinged on eliciting estimates of percentiles of an unknown distribution in a bid to address the sparsity of data attendant in risk analysis of rare events. They develop a Bayesian-based method where the parameters of a log-normal fragility curve are allowed to vary and use a state-of-knowledge distribution to describe the variability. On the premise that each pair of values of the parameters define one fragility curve, they derive a family of curves such that the probability of a particular curve being the true curve is the equal to the probability of the pair of parameter values that define it. Using Bayes Theorem, they derive the state-of-knowledge distribution in two-dimensional space of the parameters incorporating expert opinion as evidence.

Similar to the ascription of anomaly and failure data, another cornerstone of the proposed methodology is the use of expert opinion to draw the parallels between an existing, in service or retired product and a dissimilar concept product. Such a concept is not novel and in the next few paragraphs some existing works in the literature are highlighted.

Bedford et. al's *Expert Elicitation for Reliable System Design* is a review of the use of subjective expert judgment methods to assess reliability in the design process. Citing research in experimental psychology, Bedford et. al. state that accurate subjective probabilities are unobtainable by asking someone to provide a probability number, prompting the need for an elicitation process. They further state that most research into elicitation has been focused on the reduction of bias – motivational, cognitive, anchoring, and availability [28].

In their paper *Combining Probability Distributions from Experts in Risk Analysis*, Clemen and Winkler explore mathematical and behavioral approaches to combination or aggregation of probability distributions obtained from experts, [30]. The authors describe mathematical aggregation as consisting of analytic models that operate on the individual probability distribution and range from measures such as arithmetic and geometric means of probabilities to procedures based on axiomatic approaches.

Ayyub [31] provides a comprehensive overview of the use of expert elicitation and the increasing need for its use in scientific investigation in his book, *Elicitation of Expert Opinions for Uncertainty and Risks*. He cautions against the attendant pitfalls if the biases introduced by personal and group experiences are not adequately addressed. Cook [32] provides a survey of literature on the use of expert opinion in various science disciplines. His book, *Experts in Uncertainty: Opinion and Subjective Probability in Science*, provides insight on the definition of an expert, the representation of an expert's uncertainty, the determination of the value and quality of an expert's opinion, and how multiple expert opinions may be combined. Cook notes the importance of using a mathematical basis for the incorporation of expert opinion in science and suggests three well-known methods; classical, Bayesian, and psychological scaling.

2.8 Closing

The diversity of literature that address the many aspects of risk analysis in general, and the use of historical data, within the constraints of limited design information for attribute estimation, in particular, is indicative of the relevance of this field of research. The gap is the nonexistence of a unifying framework that coalesces the elements of risk assessment in a comparison-based approach while enabling a quick and agile analysis effort that generates meaningful, trustworthy results. Reiterating the core of our objective; the use of partially relevant data, this research effort seeks to bridge that gap by providing such a framework.

3 System Characterization and Similarity Quantification Methods

There is a tangible dissimilarity between a white, cotton, dress shirt manufactured in Australia and a black, wool, sweater manufactured in New Zealand with respect to the garment's cooling performance in hot weather. Conceptually, quantification of the dissimilarity is possible given a discrete set of dimensions along which to compare the garments;

- material properties - cotton versus wool
- color - white versus black
- environmental conditions - Summer versus winter
- manufacturing process - Austrialian standards versus New Zealandan standards
- the wearer's body type - Lean versus obese
- the wearer's perception of thermal comfort - High tolerance versus low tolerance

The conclusion from the foregoing is that entities under comparison must be at least partially characterized, and a context of comparison must be defined for there to be a degree of reasonableness associated with the similarity or dissimilarity measures. Whether considering evolutionary variants of a product or species, or two completely unrelated objects, it is also rational to expect that uncertainty about any estimated measures of similarity increases given fewer pertinent dimensions and fewer identifiable common characteristics.

While there maybe general agreement that there is always some relative measure of similarity or difference between any two items, the challenge lies in actually quantifying and measuring it. Before we present the theoretical foundation and methods brought to bear in our approach on quantifying similarity and characterizing entities, let us establish some nomenclature and general rules with regards to measurements.

3.1 Scales of Measurement for Characteristics, Attributes, and Features

Information and knowledge required to implement our methodology, have to be placed in the appropriate data category. Understanding of the scale of measurement for any discrete dimension of comparison will facilitate the comparison process. To compare white versus black garments with respect to thermal performance on sunny day, one may look to the reflective properties of colors. We may then draw some conclusions based on the information encoded in the measure of reflectivity.

In general, scales of measurement describe the nature of information within the numbers assigned to variables and comparators [33]. We provide a very brief discussion on the various scales to further lay the groundwork for the invesitgation of similarity quantification.

3.1.1 Nominal Scale

The nominal scale is a scale of measurement in which items are labeled using numerals. The numerals may indicate membership to a class or be unique identifiers of individuals within a class [33]. The important statistic from nominal data is the number of instances of a class or a member of a class; the actual value of the label is quantitatively useless as no mathematical computation can be performed on them. Nominal data, however allows the measurement of frequency of occurrence and the central tendency of a class.

A rule for using the nominal scale for evaluating records of failure is that the same label cannot be assigned to different classes or different numerals to the same class.

Examples of information on a nominal scale include: country, manufacturer, space mission sponsor agency, etc. These groupings can be used as associative weighting measures based on shared membership. The higher the number of shared nominal data between systems, the higher the similarity between the systems.

3.1.2 Ordinal Scale

The ordinal scale is an data categorization method that maintains the ordered series of relationships between entities. Rank ordering of information results in ordinal data. Ordinal data communicates relative increment or decrement of an entities position. However, the ordinal scale does not indicate the distance between consecutive values.

Ordinal scales can describe levels of performance (e.g, “poor” to “fantastic”). As applies to our model, it can be used to collect opinion data on proximity of shared attributes with respect to a measure or context of interest. More specifically, the ordinal data and the possibility of ordinal regression allow us to estimate the effects of change (evolution or decline). Given a degree of familiarity and experience with a number of comparable entities, human perceptive and cognitive abilities can very readily assign an order of preference, or importance to the group. As a matter of fact, product marketing research draws heavily on the idea of perceptual mapping based on rank-order data collected from surveys. Such perceptual maps are akin to Shepard’s psychological space.

3.1.3 Interval Scale

Interval scale allows measurement of quantitative data where no true zero value can be determined. As a consequence, interval scales typically have zero points specified as a matter of convention. Examples of measurements on the interval scale include, time and temperature. It is then a rather trivial exercise to quantify the similarity of information presented on an interval scale. Given two temperature readings, basic arithmetic reveals the difference. In implementating of our methodology, we leverage interval data when available and when absolutely pertinent to the metric of interest.

3.1.4 Ratio Scale

Ratio scale is the most complete scale measurement because it allows determination of the four relational measures; equality, rank-order, equality of intervals, and equality of ratios [33]. All statistical measures are applicable in ratio scale data. The number scale, which captures the true meaning of "how many", is the most representative of the ratio scale. Knowing "how many" or "how much" allows answering the fundamental question of similarity, "how close".

Before closing on scales of measuresment there are, two distinctions to be made within ratio scale; fundamental ratio scale data and derived ratio scale data. Fundamental ratio measurements include, length, weight, electrical resistance, etc, while derived ratio measurements address density, force, and elasticity. [33]. The latter are derived because they contain the inherent relationships between fundamental measurements. As with interval scale data, ratio scale information when available, and pertinent to the context of comparison provides an excellent ingredient to quantifying similarity.

3.2 Measures of Similarity: The Output of Inter-system Comparison

Establishing measures of similarity between entities largely depends on the scale of measurement appropriate for the dimensions and variables along which they are to be compared. Similarity measures for continuous data is a matter of comparing the metric value which communicates, for each entity, the rating of an attribute (e.g.thrust output of a solid rocket engine). For categorical comparators that are nominal or rank ordered, the idea of similarity is a more nebulous concept, requiring a more subjective and qualitative perception of attributes. However, once information has been properly categorized according to its proper scale, then comparison of entities described by the set of data can proceed. The following sections describe methods of similarity quantification for both metric data and categorical data.

3.2.1 Distance-Based Measures

Distance-based measures are typically used to calculate the distance between pairs of multivariate entities. The three most common are *Euclidean Distance*, L_2 *Norm*, the *Manhanttan* L_1 *Norm*, and the *Mahalanobis Distance*. These measures conform to the metric axioms listed below and thus satisfy the conditions for use in our methodology.

- Distance is positively defined for any *ith* and *jth* entities $d_{ij} \geq 0$
- Distance between an entity and itself is $d_{ii} = 0$
- Distance is symmetrical, $d_{ij} = d_{ji}$
- Distance satisfies the triangle inequality $d_{ik} \leq d_{ij} + d_{jk}$

By conforming to these metrics, distance measures used in our methodology for N entities can be represented in at most $N - 1$ dimensions.

3.2.1.1 Euclidean Distance - The Minkowski L_2 Norm

Given ratio or interval scales for measuring multivariate attributes, the L^2 Norm distance provides a measure of proximity of pairs in multidimensional space. Also commonly known as the Euclidean distance, this measure reflects the shortest straight line between two points. Mathematically, it is defined, Equation (3.1) as the shortest line segment between two points and is derived from Pythagoras' Theorem.

$$d_{ij} = \sqrt{\sum_{k} \left(x_{ik} - x_{jk} \right)} \tag{3.1}$$

where d_{ij} is the Euclidean distance between two entities i and j, and k is the number of dimensions. The variables X take on values x for each entity along any of the given dimensions.

3.2.1.2 Manhattan Distance - The Minkowski L_1 Norm

The Minkowski p-metric is a general class of distance measures defined by Equation (3.2) ;

$$d_{ij}(p) = \sqrt[p]{\sum_{k} \mid x_{ik} - x_{jk} \mid^{p}} \tag{3.2}$$

The Euclidean measure is a special case of the L_p measure when $p = 2$. Another case of the L_p measure is when $p = 1$

$$d_{ij}(1) = \sum_k \mid x_{ik} - x_{jk} \mid \tag{3.3}$$

Equation (3.3) is commonly referred to as the *city block*, Manhattan distance, or the L_1 norm, since it is equivalent to from point A to point B in a city with perpendicular street arrangement.

In two-dimensional space, the L_1 and L_2 norms are distinguished by the fact that around any point, the contours of equal distance and generalization are circluar for $p = 2$ and rhombic for $p = 1$ [15]

3.2.1.3 Mahalanobis Distance A separate class of distance measures from the Minkowskian L_2 and $L - 1$ norms is the Mahalanobis Distance. It is a measure of distance that reflects any inherent covariance in the data [34]. It is given by Equation (3.4)

$$d_{ij}^2 = (\mathbf{x}_i - \mathbf{x}_j)'\Sigma^{-1}(\mathbf{x}_i - \mathbf{x}_j) \tag{3.4}$$

3.2.2 Feature Matching Measures

Matching measures are appropriate when dealing with nominal scale attributes. Since distance-based measures cannot be applied to nominal attributes, the usual approach is then to match attributes. In this case, the degree of similarity is couched in terms of the extent to which entities share attributes and are derived from feature matching functions described in the following sections.

3.2.2.1 The Contrast Model

The Tversky Contrast Model is a measure of similarity introduced by Tversky [35] as part of his feature set theoretics for comparing variants among entities. To define the similarity of a to b, $s(a, b)$, Tversky establishes three assumptions of 1) matching, 2) monotonicity, and 3) independence. For matching, $s(a, b)$ is defined as a function of three arguments where s is an ordinal measure of similarity

$$s(a, b) = F(A \cap B, A - B, B - A) \tag{3.5}$$

where $A \cap B$ is the set of features common to both a and b, $A - B$ is the set of features that belong to a and not b, and $B - A$ is the set of features that belong to b not a. For monotonicity;

$$s(a,b) \geq s(a,c) \tag{3.6}$$

whenever;

$$A \cap B \supset A \cap C,$$

$$A - B \subset A - C,$$

and,

$$B - C \subset C - A$$

For independence;

$$s(a,b) \geq s(a',b') \tag{3.7}$$

if and only if

$$s(c,d) \geq s(c',d')$$

In addition to the three assumptions, Tversky includes *invariance* and *solvability*. Invariance ensures that equivalence of intervals is preserved, while solvability requires that the feature space be sufficiently populated such that similarity equations be solvable {35}.

Under all five conditions, the contrast model introduces a similarity scale S and a nonnegative scale f such that for all entities a, b, c, d in a set,

$$S(a,b) \geq S(c,d)$$

iff

$$s(a,b) \geq s(c,d)$$

$$S(a,b) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A) \tag{3.8}$$

Equation (3.8) is the matching function that defines the Contrast Model for feature matching given weighting coefficients $\theta, \alpha, \beta \geq 0$ and interval scales for f and S.

3.2.2.2 The Ratio Model The *ratio model*, Equation (3.9) expresses similarity between objects as a ratio of the measures of their common and distinctive attributes[36]. It was also introduced by Tversky and essentially normalizes similarity such that S is bounded between 0 and 1. It generalizes other set-theoretic models such as Jaccard, Dice, and Tanimoto, which differ only based on the values assigned to the weighting coefficients in the matching function.

$$S(a,b) = \frac{f(A \cap B)}{f(A \cap B) + \alpha f(A - B) - \beta f(B - A)} \tag{3.9}$$

3.2.2.3 Jaccard Index The Jaccard index, Equation (3.10) for measuring set similarity results from setting the importance parameters $\alpha = \beta = 1$:

$$Jaccard(a,b) = \frac{f(A \cap B)}{f(A \cup B)} = \frac{f(A \cap B)}{f(A) + f(B) - f(A \cap B)} \tag{3.10}$$

3.2.2.4 Dice Coefficient The Dice index, also known as the Sorensen index, Equation (3.11) for measuring set similarity results from setting the importance parameters $\alpha = \beta = \frac{1}{2}$:

$$Dice(a,b) = \frac{2f(A \cap B)}{f(A) + f(B)} \tag{3.11}$$

3.2.2.5 Tanimoto Coefficient Extension of the Jaccard coefficient to sets whose members are not resticted to binary forms yields the Tanimoto coefficient. It assumes that the sets are vectors of set members and the similarity index is given by;

$$Tanimoto(a,b) = \frac{f(A \cap B)}{f(A)^2 + f(B)^2 - f(A \cap B)} \tag{3.12}$$

The index reduces to the Jaccard index for binary set members.

The distance measures discussed above provide opportunities for similarity input to our methodology. With the proper measures of scale, and the appropriate distance quantification method, we can assess the relatedness of entities and proceed. However since we are concerned with not only attibute-to-attribute comparisons, but with more ethereal concept of applicability of historical data to new systems, we turn to the concept of evolution of entities for insight on why and how things change.

3.3 Methods-I: Evolution as Input to Similarity Quantification

Several definitions of "evolution" aptly illustrate elements of this thesis. Of the various definitions listed by the Merriam Webster Dictionary, we restate the three most relevant to our context:

- a process of change in a certain direction
- a process of continuous change from a lower, simpler, or worse to a higher, more complex, or better state
- descent with modification from preexisting species: cumulative inherited change in a population of organisms through time leading to the appearance of new forms: the process by which new species or populations of living things develop from preexisting forms through successive generations

Merriam-Webster, additionally, quotes Stephen Jay Gould on evolution, "the scientific theory explaining the appearance of new species and varieties through the action of various biological mechanisms".

Evidence of evolution is manifested through observable changes in the characteristics and behavior of species and products. Evolutionary biology, a sub-discipline of biology that studies evolutionary processes including the descent of species, has given rise to formalized methods for inter- and intra-species comparison such as phylogenetic comparative methods (PCMs) for studying trait evolution [37]. According to [37], PCMs include ancestral state reconstruction, phylogenetically independent contrasts (PICs), and phylogenetic generalized least squares (PGLS). Such formal methods can be leveraged for studying the evolution of engineered systems and making inter-product and intra-product evolution comparisons. However while PCMs are aimed at elucidating the mechanisms at the origin of diversity of species [38], our focus in this thesis is on determining the degree of similarity between engineered systems by subjectively assessing the degree to which they have evolved.

From the foregoing, one can regard majority of engineered systems as the result of the evolution of an existing or previous design. On this premise, the degree of change or evolution from precedent to concept may provide a measure of the similarity between both. As a matter of fact, biologists under the same premise quantify similarity between species, whether physical or behavioral. Comparison of species in various stages of evolution is possible when the evolutionary path is known. For example, human evolution is typically illustrated as progression from hominids to Homo sapiens as shown in Figure 3.1. From comparing these inter-species variants, measures of similarity with respect to any chosen characteristic can be quantified. For example, hominids can be compared with homo sapiens along several metric (i.e. measurable), or non-metric/categorical dimensions of comparison , uprightness, height, intelligence, average weight, strength, etc. By evaluating variants along particular dimensions of comparison a metric for similarity, whether metric or categorical, can be ascertained.

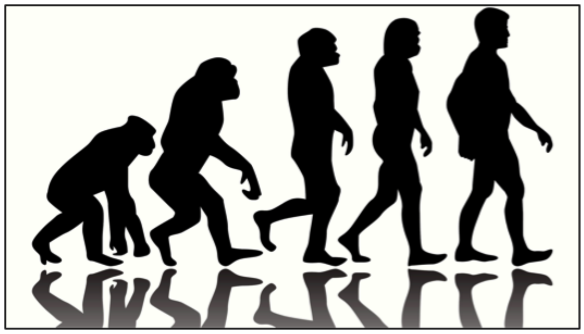


Figure 3.1
Human Evolution (Source: http://kingofwallpapers.com/evolution.html)

In reliability engineering, system designs are modified to address known failure modes and perhaps reduce the probability of occurrence in newer models and future variants. Design changes however, may not always to be aimed at improving the reliability of a previous design; safety, performance, cost, and product appeal are typical considerations that factor into the decision to modify systems. These other considerations are possible dimensions for comparison of engineered systems but in this thesis, the primary concern is with product evolution that impact the reliability of a system.

Where changes due to other considerations explicitly contribute to the failure behavior of the system, such contributions are accounted for in probabilisitc failure analysis. Nonetheless, a prerequisite for using evolution as input to quantification of similarity is having a clear picture of the evolutionary path a product has taken such that there is a physical, visual, or intrinsic attribute, and a set of behavior-influencing external circumstances that make differentiation of variants possible.

To set the stage for quantifying the similiarity of attributes between engineering system, consider the following analogy between biological system evolution and the evolution of engineered system.

3.3.1 Biological Analogy to Evolution of Engineered Systems

Biological evolution, triggered by adverse events, reflect adaptation of the evolving species for the purpose of finding the most optimal attributes for its continued existence and movement towards a higher state of being. There are five mechanisms of biological evolution: mutation, genetic drift, natural selection, gene flow, and non-random mating [39] of which, the natural selection process is highly analogous to the process of design or technological evolution in an engineering setting.

According to biologists, evolution by natural selection occurs “when the environment exerts a pressure on a population so that only some phenotypes survive and reproduce successfully”[40]. This is analogous to the engineering design process where deliberate choices are made by system developers and design engineers that affect the behavior of an engineering system. In designing for reliability, design choices are intended to mitigate failure processes previously documented for existing variants. This design selection process by human actors who are reacting to adverse events like failures of previous variants is analogous to the external influences on biological species. While biological species possess phenotypes which “refer to all the manifold biological appearances, including chemical, structural and behavioral attributes, that we can observe about an organism but excludes its genetic constitution” [41] that evolve via natural selection, engineered systems possess design characteristics, material properties, and functional attributes affected through human intervention that dictate their performance.

Design choices made during the evolution of an engineered system are manifested in observable changes in the system’s analogue to species’ phenotypes. A summary of the foregoing is presented in the following bullets to explicitly state the underlying connection between human experiential learning and engineering evolution:

- Engineering systems evolve because humans interfere and impose a natural selection of attributes through the design process
- The motivation to evolve designs may be derived from observations of past failure, as implied by the learning and adaptation feedback mechanism inherent in all sentient forms.
- System designs evolve in response to human experiences, and invariably, human experience changes as system designs evolve

Evolution of engineered systems as an outcome of human experience raises the possibility of extending theories of stimulus-response and generalization that have thus far only been applied in psychometry, psychology, and behavioral sciences to this thesis. We make this extension by regarding manifestations such as the observed failures and anomalous behavior (human experience) of precedent systems as the **response**, and the system design with its underlying failure modes or failure-susceptible elements as the **stimulus**. As is the case in [12], and [13], we propose a nuanced stimulus-response relationship where relative system design is the independent variable that predicts an observable response. The nuance lies in the interpretation of “relative” in “relative system design", implying that the independent variable is a quantifiable degree of similarity between an evolved system and its precedent.

3.3.2 Evolution of Engineered Systems

In the previous section, the case has been made that engineered systems indirectly evolve as a result of a human-induced natural selection process. Examples of the evolution of engineered systems are all around us; model year-to-model year changes for automobiles reflect shifting trends in technology and societal preference. Also, spacecraft platforms evolve to suit mission needs based on shifting science focus, national security objectives, and commercial factors.

Other evidence of quantifiable evolution of engineered systems is abundant as we see elimination of obsolete or lower forms of technologies across a variety of engineering sectors; for example computing systems are updated to take advantage of faster processing speeds, increased memory and storage capabilities, and new materials with improved properties are introduced.

These changes, while largely heralded as improvements, sometimes force a rethinking of engineering processes due to subsequent introduction of new failure modes in systems. These new failure modes, not previously accounted for, once experienced become a catalyst for the learning and adaption feedback process. Notwithstanding the impacts of change, the fact remains that a measure of any such change can be leveraged in predicting the "response" of the evolved system, bearing in mind the description of "response" as the manifestation of failure or anomalous behavior.

To measure the degree of change, or evolution, one not only has to have two or more systems or circumstances, but also have an established basis or dimension for the comparison, hence the need for a classification scheme that categorizes the so-called phenotypes and characteristics of the system. In the next section, we define a hierarchical structure for describing systems to facilitate a context-based system-to-system comparison.

3.4 Methods-II: Taxonomy for Characterization of Engineered Systems

Through the use of a taxonomic approach that establishes the common framework for comparison of systems, we define a vocabulary for a finite set of system components and their functional and structural relationships.

3.4.1 Hierarchical Taxonomy for Inter-system Comparison

Taxonomy is the process of grouping species according to categorized characteristics. Engineering systems typically have an invariant set of attributes that allows identification of their essential subsystems such that taxonomy can be adapted for the purpose of grouping systems. Every car has tires, an engine, a body structure, a steering wheel, and so on. These are universally accepted as "parts of" the automobile system. The specific type of tire or engine is a differentiator and provides a basis for comparison between cars along that characteristic. Universally accepted "parts" of a system provide an ideal starting point for a comparison framework. Conceptually, one can then compare the specific instances, or "types", within the "parts" to ascertain the degree of similarity.

There are a variety of options available to designers when making decisions on type; manual versus automatic transmission, 4-cylinder versus 6-cylinder engine, spin-stabilization versus three-axis stabilization, mono-propellant versus bi-propellant propulsion system. These options are variables that determine the taxonomic contribution with respect to the context of comparison. Further decomposition of the "types" in turn re-categorize them as "parts", albeit at a lower level of the system's overall structure.

When adequately specified for an engineered system such as a spacecraft, a hierarchical taxonomy establishes the common framework for comparison of such complex systems. By defining a vocabulary for a finite set of spacecraft attributes, their functional and structural relationship, a taxonomy would provide the common ground for evaluating spacecraft design similarity upon specification of the "types" inherent within the taxonomy. Furthermore, by representing the system under development in a manner such that the relationships between its various subsystems, components and parts are maintained and clearly delineated as the design matures, a failure model of the system in the form of a reliability block diagram or a fault tree can be developed. Such a model reflects the configuration of those functional relationships between the various elements of the system to the extent that quantification of their contributions to the reliability of the system can proceed.

Taxonomies are a classification scheme that can be used to categorize information [1] such as the features of a system and they consequently provide an approach for establishing the common framework for comparison that we seek. Figure 3.2 is an example of a space mission taxonomy that illustrates the hierarchical relationship of subsystems common among most variants of spacecraft. The figure illustrates the parts of a space mission, where only the color-shaded systems are expanded to maintain compactness. The complete space mission hierarchy is included in the appendices. With further insight into a specific spacecraft's design, it is possible to generate a fault tree or reliability block diagram given this general taxonomy.

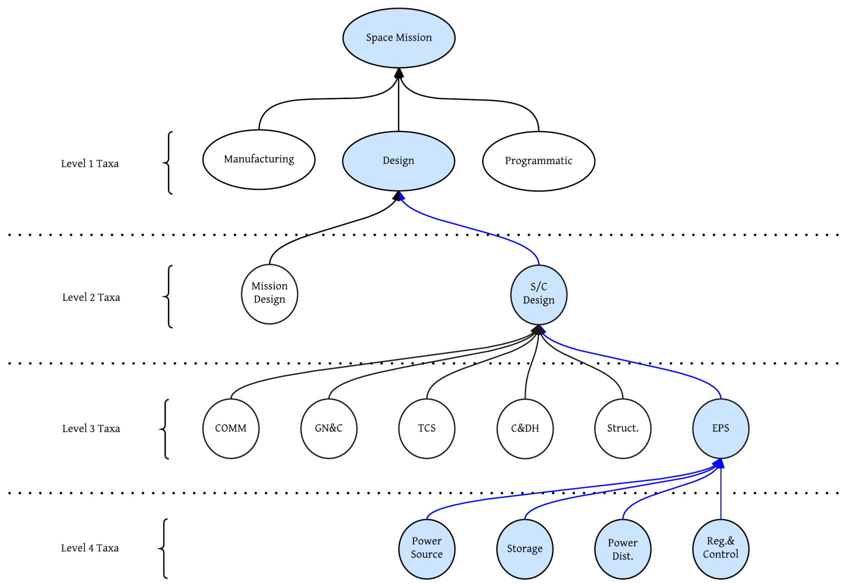


Figure 3.2

Spacecraft Taxonomy

3.4.2 Context of Comparison: Failure Contributors And Mitigators

Evaluation of similarity between two systems for the purpose of inferring the behavior of one system based on observations of another should be conducted in context with the stimuli that elicit the response behavior. Our similarity evaluations are in the context of failure- and anomaly-inducing attributes or success-enhancing and mitigating attributes of the systems. To this end, we compare attributes that either exacerbate or mitigate failure modes. For example, if the thickness of thermal insulation is uniquely indicative of the effectiveness of mitigating a temperature-induced failure, then one can compare the insulation thickness in both systems as an indicator of effectiveness against temperature-induced failure and conclude that the design with higher insulation properties is better all else being equal.

We introduced the notion of a "taxon variable" as an instance of a taxon. For context-based comparison, the taxon variable should contribute to, mitigate, or be susceptible to the failure modes of its taxon. As in Figure 3.2, i, indicates the hierarchical level and the taxon variables for any taxon in $Level_i$ include the variables for the lower level taxa associated with it.

3.5 Methods-III: Use of Ordination Methods to Develop Pseudo-spatial Configurations of Engineering Systems

Ordination techniques have a wide range of applications, from plant community-species studies in ecology and spatial models of voting in political science to genetics and psychometrics. None of these applications extend the use of discovered relative distances between pairs as a means of estimating the applicability of data between pairs of variants.

Ordination methods refer to multivariate or multidimensional analysis techniques which conform a set of entities with *N* variables to at most an *N-1* dimensional spatial configuration such that the axes of the space reveal any underlying patterns inherent in the original data. The techniques are commonly applied to data sets with numerous attributes. Consider a one-dimensional spatial arrangement of variants of a system in which the single context of comparison is the weight of the variants. These variants can be ordered according to their weights yielding a one-dimensional linear configuration of the variants. Adding a second dimension which represents another observable attribute of the variants, color, provides another axis along which to order the variants. By adding a dimension for every observable attribute, the resulting configuration of variants is a high-dimensional space from which one cannot deduce any meaningful pattern, structure or relationships. Nonmetric Multi-dimensional Scaling (NMDS) produces a spatial configuration which retains an "all dimensions considered" ordering of variants in as few dimensions as possible. Further analysis of the spatial solution reveals the principle dimensions along which the variants have been structured. By discovering an underlying spatial configuration of the entities in a low-dimensional space, a measure of quantitative distance (such as Euclidean distance) between pairs of entities can be obtained.

Through a combination of metric and subjective measures of proximity, we create the spatial configuration of entities with respect to attributes of interest, and consequently establish a mechanism for not only quantifying similarity but also for relating the quantified similarity to the degree of data relevance. Such a mechanism in ideal circumstances has the following qualities [42]:

1. Recovers gradients without distortion.
2. Reveals existent clusters in the ordination solution.
3. Does not produce nonexistent clusters.
4. Yields consistent results every time for a given set of entities.
5. Relates entity similarity to proximity in ordination space.
6. Separates signal from noise

Several methods of ordination have been proposed and have widespread use. These methods can be grouped into two broad categories; indrect gradient analysis and direct gradient analysis [42] as shown in Table 3.1.

Table 3.1
Ordination Methods

Indirect Gradient Analysis		Direct Gradient Analysis
Distance-based Methods	Eigen analysis-Based Methods	Linear and Unimodal
Polar ordination, PO (Bray-Curtis ordination)	Principal Components Analysis	Redundancy Analysis (RDA)
Principal Coordinates Analysis (Metric multidimensional scaling)	Correspondence Analysis CA(Reciprocal Averaging)	Canonical Correspondence Analysis (CCA)
Nonmetric Multidimensional Scaling (NMDS)	Detrended Correspondence Analysis	Detrended Canonical Correspondence Analysis
	Principal Components Analysis	

Distance-based ordination techniques such as PCoA or MDS, NMDS, rely on a distance matrix as input. Not to be confused with the resultant ordination distances obtained from the solution, the input distance matrix is generated from observed or subjective measures of proximity making distance-based methods suitable for our purpose. PCoA/MDS methods maximize the linear correlation between the distances in the input matrix where as NMDS maximizes and maintains the rank order of distances. This feature of NMDS relaxes the requirement for using input distance matrices that are based on explicit metric measures of proximity since rank order preferences and judgments can be easily generated using an ordinal scale.

3.5.1 Metric Multidimensional Scaling

Metric Multidimensional Scaling or Classical Multidimensional Scaling also commonly known as Principle Coordinates Analysis is a multidimensional scaling technique that is based on distance matrices derived strictly from metric distances with no confusion as to the interpretation of "distance". It is a method that produces a spatial representation of the relative position of a number of objects based on an input matrix of distances called a proximity matrix that directly arise from empirical measurements or a correlation matrix. The method tries to preserve the original metric distances in the proximity matrix.

To illustrate the idea behind the technique, consider a data set consisting of distances between pairs of objects pulled from a list of four objects, A, B, C, D. By iteration we seek to find a multidimensional arrangement of the objects that results in the original distances between pairs. If a priori, we have coordinate data (Table 3.2) on the four objects in two-dimensional space with an x- and y-axis, the spatial configuration is represented by a scatterplot of the cordinates as shown in

Figure 3.3

Table 3.2

Coordinates of four objects in 2-dimensional space

Object	X-coordinate	Y-coordinate
A	2	2
B	3	4
C	4	3
D	1	6

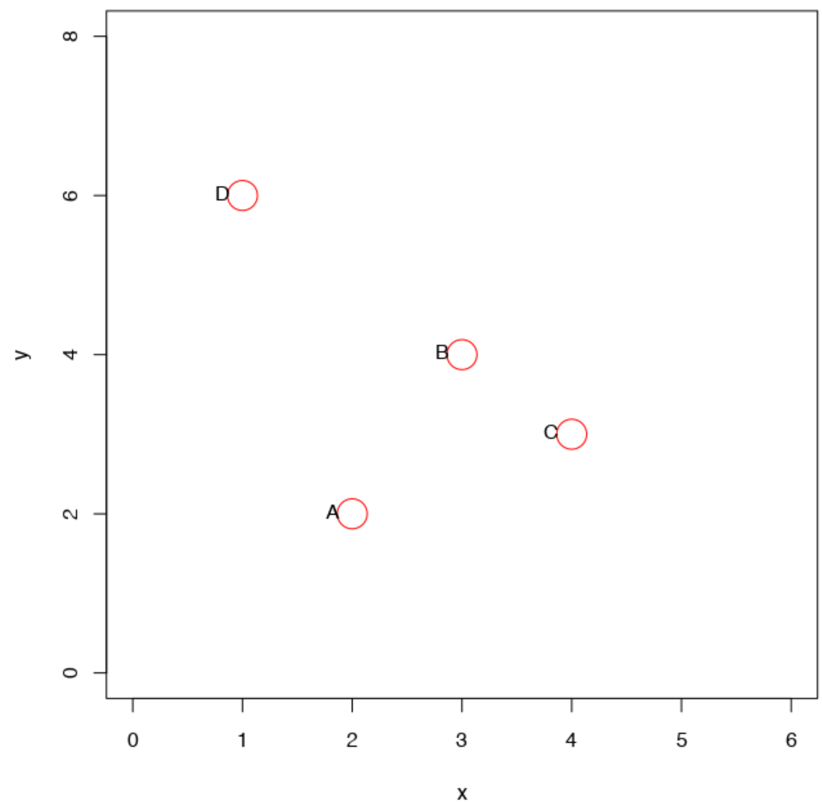


Figure 3.3

Scatter Plot of 4 objects in 2D Space

From the plot, we can assess proximity of the objects, where the distance between pairs d_{ij} is given by the Euclidean distance where d is the number of dimensions and d_{ij} is the distance between the *ith* and *jth* objects.

$$d_{ij} = \sqrt{\sum_{n=1}^{d} (x_{in} - x_{jn})^2}$$

The resultant symmetric matrix of interpoint distances is given in Table 3.3:

Table 3.3
Proximty matrix of 4 objects in 2D space

	A	B	C	D
A	0			
B	2.236068	0		
C	2.236068	1.414214	0	
D	4.123106	2.828427	4.242641	0

The objective in metric MDS is to recreate the scatterplot starting with only a matrix of interpoint distances, however this task is complicated by the fact that ordination techniques, as discussed earlier, can produce results in up to *N-1* dimensions where *N* is the number of objects. Additionally, the multiple solutions can yield the same distances but with different configurations; for example, rotating the scatterplot maintains the distances but results in shifts in the coordinate of the objects, implying that different sets of x-coordinate and y-coordinate data, as in Table 3.2 could have produced the same proximity matrix.

In furtherance of our objective of determinining similarity between objects through ordination, we consider the situation where the x- and y-axes of the scatterplot each represent some continuous measure of attribute values for the objects. On this premise, one can conclude that B and C are most similar among all pairs along both axis and as such have a higher degree of similarity. Given the ordinated distances, we can find the monotone function of distance which would relate the objects in the ordination solution. For the case where the input matrix is identical to the ordinated distances, the monotone function would have unit slope. Visualization of the monotone relationship between observed dissimilarity and the ordinated distances is called a Shepard plot; Figure 3.4 shows the resuling Shepard for our simple four object metric ordination.

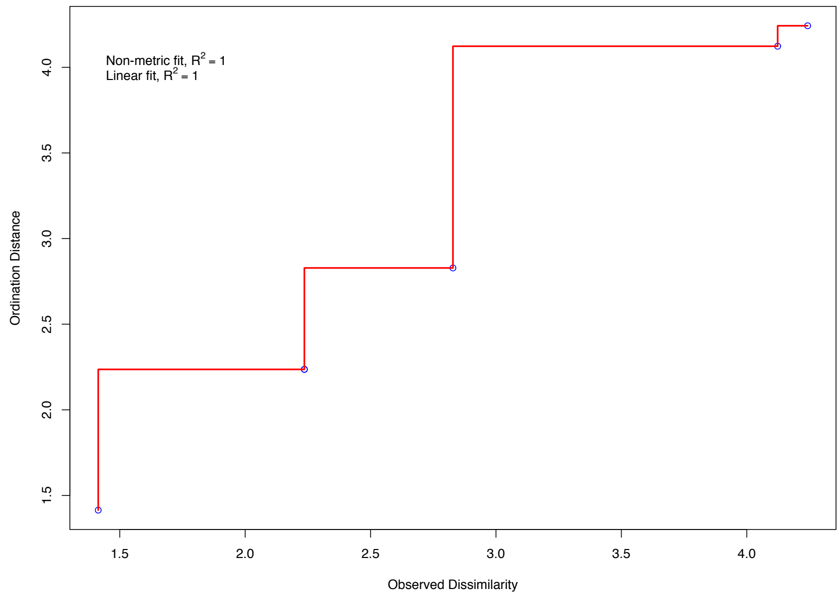


Figure 3.4

Shepard plot of four-object coordinate data

3.5.2 Nonmetric Multidimensional Scaling

As defined earlier, NMDS method yields ordination based on a distance or dissimilarity matrix by resolvng a number of points into a prespecified number of dimensions while maintaining, as much as possible, the rank-ordered pairwise or inter-point dissimilarities between the points. Input to the ordination process is based on results of a survey where respondents provide their subjective assessment of the proximity between pairs of entities. These pairwise measures of proximity can then be rank-ordered regardless of the actual distances. The measure of departure from the inital rank-order is called the stress [19] of the solution. Mathematically, Kruskal defined stress as:

$$Stress = \sqrt{\frac{\sum \left(f\left(d_{ij}\right) - D_{ij}\right)^2)}{\sum D_{ij}^2}} \tag{3.13}$$

$f(d_{ij})$ is the optimal monotonic transformation of the dissimilarities which minimizes the ordination stress and D_{ij} are the interpoint distances determined from the observed dissimilarities, δ_{ij}. Monotonic transformation is a least-squares smoothing process accomplished through a monotone regression and results in transformed distances such that d_{ij} and δ_{ij} have the same rank order. In NMDS, our aim is to uncover a configuration such that the D_{ij} and the δ_{ij} have the same rank order. For example, assuming the interpoint distance between two entities ranks fourth in the set of subjective δ_{ij}s, then they should also rank fourth in the ordination solution. Points, d_{ij} on the monotonic curve $f(d_{ij})$ are the prediction values from regressing D_{ij} on δ_{ij} and the goodness of the fit is measurable by comparing the D_{ij} to d_{ij}. Kruskal provided guidelines based on empirical data for assessing goodness of fit using stress values.

Table 3.4

Kruskal's Guidelines for Assessment of Stress

Kruskal's Stress	Goodness of Fit
0.2	Poor
0.05	Good
0.00	Perfect

While NMDS solutions are obtainable in up to $N-1$ dimensions for N points, the penalty for a lower, and often, more desirable solution is a higher stress. Figure 3.5 is a plot of stress as a function of dimensionality, referred to as scree plots. It provide a useful visualization for evaluating the miniminal stress for a desired number of dimensions.

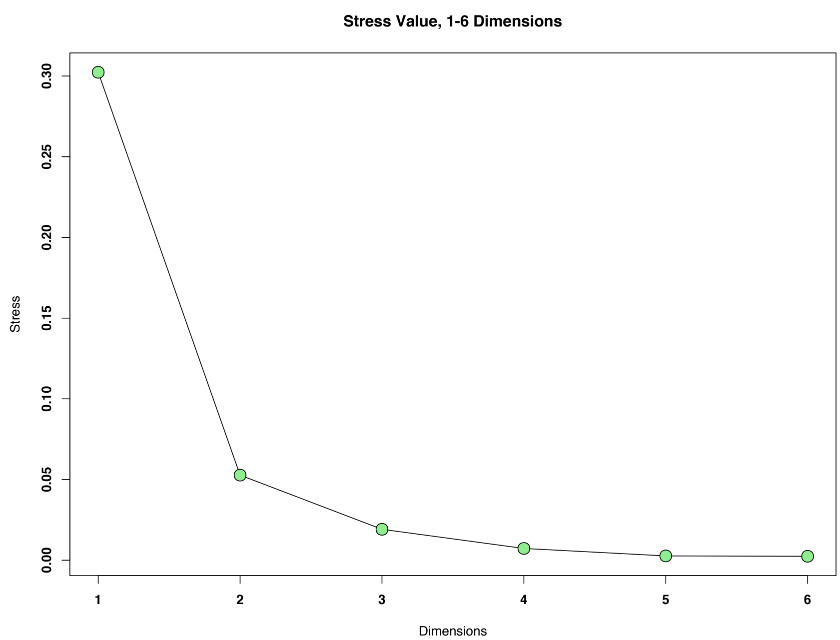


Figure 3.5

Scree Plot

3.6 Methods-IV: Analysis of Spatial Data through Kriging Interpolation

Given that we have successfully developed a spatial configuration which communicates the proximity of the entities of interest, our next task is to infer properties of one entity from those of another as a function of their spatial proximity and the parameters of the spatial field.

By ordinating the proximity measures elicited from the comparison of engineered systems via NMDS, [19] [17], we construct a pseudo-spatial arrangement of designs which lends to spatial analysis and invocation of Tobler's "First Law of Geography"; "Everything is related to everything else, but near things are more related than far things" [43]. This law has utility in various applications including geology, soil sciences, meteorology, political science, etc. Our thesis proves the viability of this theory of relatedness as a function of distance when considering spatial representations and spatial data obtained based on rank-ordered, intuitive measures of proximity.

Akin to Shepard's psychological space [15], the pseudo-spatial arrangement of engineered systems exhibits the invariant relationship between response generalization and the distance between locations. However, while Shepard establishes the single parameter exponential function as that invariant relationship between generalization and distance, the so called generalization gradient, we seek a more descriptive relationship for capturing spatial correlation. To this end, we draw from the field of geostatistics and specifically, the analysis of spatially correlated data using point-refernced data models.

Kriging is a method of spatial interpolation, also known as Gaussian process regression, for estimating variables at an unmeasured location from values at surrounding locations. The approach, is based on the work of Daniel G. Krige, the South African miner, who devised the method for estimating the distribution of gold based on samples from few locations. Kriging yields optimal interpolation of the target variable based on regression against actual observatons weighted with respect to the implict, field-specific, spatial correlation of locations.

Fundamentally, Kriging interpolation methods calculate the metric of interest at the target location as a weighted sum of values from neighbouring locations. Determination of Kriging weights, consistent with Tobler's Law, is based on monotonically decreasing functions that ensure decreasing weight as a function of spatial proximity. However, the actual parameterized weighting function is derived from the point-referenced data of the spatial field. As with any interpolation method, Kriging results in good estimates of the unknown of interest given a well characterized spatial field, i.e., underlying parameters of the field parameters are estimated with absolute certainty in the face of infinite data. Furthermore, Kriging results typically underestimate the high end of the unknown of interest and overestimate the low end. This behavior is also consistent with traditional averaging techniques.

We present a brief overview of the Kriging estimation process and different types of Kriging.

3.6.1 Kriging

Kriging methods are derived from minimum mean square error prediction. Following derivations presented in the literature, specifically [44], we let $\boldsymbol{Y}$ and T represent vectors of random variables that take on observed values and a random variable we wish to predict from the observations of $\boldsymbol{Y}$ respectively. Point estimation of observations of T is obtained from any function;

$$\hat{T} = f(\boldsymbol{Y})$$

and the mean square prediction error MSE of $\hat{T}$ is given by:

$$MSE(\hat{T}) = E[(\hat{T} - T)^2]$$

Where the expectation, $E[\cdot]$, is with respect to the joint distribution of T and $\boldsymbol{Y}$. The form of the Kriging estimator which minimizes $MSE(\hat{T})$ is the expectation of T conditional on $\boldsymbol{Y}$:

$$\hat{T} = E[(T \mid \boldsymbol{Y})]$$

Resulting from Equation (3.14) [44], is the relationship:

$$MSE(\hat{T}) = E_Y[Var(T \mid \boldsymbol{Y})] \tag{3.14}$$

where $Var(T \mid \boldsymbol{Y})$ is the prediction variance. Its value, given the observed values of $\boldsymbol{Y}$, estimate the mean square error of estimates of T. Equation (3.14) provides the basis for the minimum error kriging estimator.

Assuming that observations in $\boldsymbol{Y}$ are descibed by a stationary Gaussian process, such that:

$$\boldsymbol{Y}(\boldsymbol{s}) = \mu(\boldsymbol{s})\boldsymbol{\beta} + W(\boldsymbol{s}) + \boldsymbol{\epsilon}(\boldsymbol{s}) \tag{3.15}$$

where $\boldsymbol{\epsilon} \sim N(\boldsymbol{0}, \tau^2)$ is the error term, $\mu(\boldsymbol{s}) = \boldsymbol{S}^T(\boldsymbol{s})\boldsymbol{\beta}$ is the mean[2], and $W(\boldsymbol{s}) \mid \sigma^2, \phi \sim N(\boldsymbol{0}, \sigma^2 H(\phi))$ accounts for spatial correlation. We leverage the general results from multivariate normal theory in predicting observations at locations in $\boldsymbol{T}$ given spatially correlated observations in $\boldsymbol{Y}$.

From multivariate normal theory, if

[2]The mean is a function of spatial covariates $\boldsymbol{S}$ and regression coefficients, $\boldsymbol{\beta}$

$$\begin{pmatrix} \boldsymbol{Y}(s) \\ \boldsymbol{T}(t) \end{pmatrix} \sim N\left(\begin{pmatrix} \mu(s) \\ \mu(t) \end{pmatrix} \quad , \quad \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$$

then the joint distribution of two random variables $\boldsymbol{Y}(s) = (Y(s_1), ..., Y(s_n))'$, and $\boldsymbol{T}(t) = (T(t_1), ..., T(t_m))'$ given a collective set of distribution parameters $\boldsymbol{\theta}$, is given by Equation 3.16

$$(\boldsymbol{Y}(s), \boldsymbol{T}(t) \mid \boldsymbol{\theta}) \sim MVN\left(\mu \mathbf{1}_{m+n} \quad , \quad \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right) \tag{3.16}$$

Furthermore, the conditional distribution of unobserved location values, $\boldsymbol{T}(t)$, given the observed location values, $\boldsymbol{Y}(s)$, and model parameters $\boldsymbol{\theta}$, is given by Equation (3.17);

$$(\boldsymbol{Y}(s), \boldsymbol{T}(t)) \mid \boldsymbol{\theta}) \sim MVN_m(\mu_{2.1}, \Sigma_{2.1}) \tag{3.17}$$

where the mean is, $\mu_{2.1} = \mu \mathbf{1}_m + \Sigma_{21}\Sigma_{11}^{-1}(\boldsymbol{Y}(s) - \mu \mathbf{1}_n)$, and the variance is, $\Sigma_{2.1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$

Equation (3.17), extended to pseudo-spatial maps derived from ordinated, subjective measures of proximity between precendent and conceptual systems informs our Bayesian analysis framework.

3.6.2 Simple Kriging

In Simple Kriging, it is assumed that the mean of a spatial process is constant over the entire spatial field with no covariate effect resulting in:

$$\boldsymbol{Y}(\boldsymbol{s}) = \mu + W(\boldsymbol{s}) + \boldsymbol{\epsilon}(\boldsymbol{s}) \tag{3.18}$$

3.6.3 Ordinary Kriging

In Ordinary Kriging, the mean of the spatial process is assumed to be constant only in the immediate neighborhood of each observed location s_i and meter the unobserved locale means by constraining the kriging weight to sum to 1. The least squared error expression, Equation (3.14), (i.e the linear predictor of the mean that minimizes the expectation of variance);

$$\boldsymbol{Y}(\boldsymbol{s}) = \mu(\boldsymbol{s})\boldsymbol{\beta} + W(\boldsymbol{s}) + \boldsymbol{\epsilon}(\boldsymbol{s}) \tag{3.19}$$

3.6.4 Universal Kriging

Universal kriging is similar to ordinary kriging except that in addition to the local trend in the mean, a global trend based on coordinates of each loaction is fit to the overall spatial field. This is the same as in Equation (3.15). Recall that the mean of the process is defined as a funcion of spatial covariates and regression coefficients.

$$\mu(\boldsymbol{s}) = \boldsymbol{\beta_0} + \sum_{m=1}^{M} \boldsymbol{\beta_m} \boldsymbol{s} \tag{3.20}$$

4 Towards Analysis of Pseudo-spatial Models of Comparable Systems

4.1 Overview

As shown in Figure (4.1), the analysis framework consists of two broad aspects. First is a characterization and comparison phase in which Methods I and Methods II are used for defining the systems intended for comparison. This is with a view to identifying the attributes that are pertinent to the target of inference. In this phase, categorical and numerical proximity measures elicited from expert comparison provide input to Methods III used in creation of the perceptual map of the systems. Implicit in the use of these methods are some general assumptions; 1) a general taxonomy is adequate for characterization of a system, including concepts, 2) the similarity between entities is symmetric, 3) the interpoint distance between systems in the generated spatial map conforms to metric axioms discussed in Chapter 3.

Following characterization of the systems, is a knowledge integration and inference phase. In this phase we leverage spatial modeling methods, Methods IV, and Bayesian principles to develop the posterior probability densities of the spatial process parameters with which to estimate the metric of interest

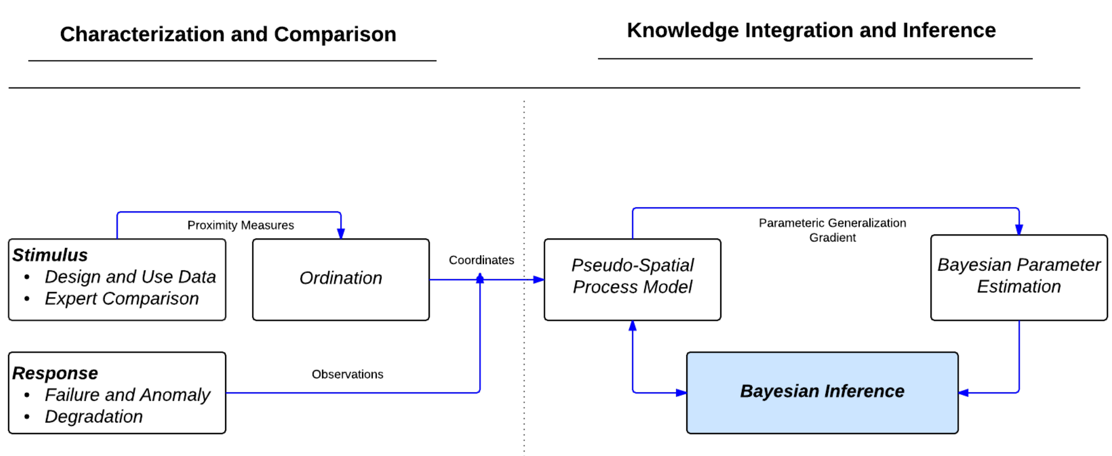


Figure 4.1
Overview of methodology

4.2 Data Sources

Risk and reliability models are representations of reality through which various contributors to a system's behavior can be studied. However, due to the aleatory uncertainty associated with the models representing the reality of interest, understanding of those contributors is often cloudy. Our methodology attenuates the aptness of the chosen models through an adjustment derived from a measure of similarity between entities. To exercise the methodology, we require three distinct types of input:

1. A record of demonstrated behaviors ascribable to a specific, existing or, previously existing, system in a defined environment. These behaviors may include failure rate information, degradation data, lifetime data, or dichotomous response-on-demand data
2. A characterization of the existing or, previously existing, system and it's particular operational environment together with a conceptual design of a system which belongs to the same general class and potential use environments. These characterizations may include design information, descriptions of environments and use scenarios. Additionally, a taxonomy of the general class of such systems that elucidates all fundamental attributes required for the system's intended function. These may include a functional descriptions of categories within the general class
3. An expert trained and knowledgeable in the design, engineering, and use of the general class of such systems and equiped to provide comparisons between concepts and precedents within a particular, given context (reliability, affordability, manufacturability, etc.)

4.2.1 Record of Anomalous Behavior

We define a precedent system as one developed for the same general purpose as the concept. Existing automobile models may provide precedence for new models. In aerospace, existing or retired spacecraft platforms may serve as precedence for new spacecraft. To implement the proposed methodology, we need to determine the body of evidence, which represents the historical failure information and serves as the precedent's record of demonstrated behavior. Collection of failure and anomaly data is common practice across various industries. These data are typically stored in warranty databases and industry-required failure reporting and corrective action databases.

In an aerospace application of the methodology, on-orbit anomaly and failure information on spacecraft missions and major subsystems of a spacecraft provide input towards quantifying the demonstrated reliability of existing platforms. These data can inform design choices as new systems are developed. Failure data required to implement the methodology are obtainable from numerous sources. For spacecraft data, one repository of satellite information, The Satellite Encyclopedia (TSE), provides comprehensive data on various platforms, thus allowing analysis of performance trends across multiple missions. TSE is a subscription-based web service based in Europe and owned by Tag's Broadcasting Service. Other sources include; NASA Goddard Spaceflight Center's Spacecraft On-orbit Anomaly Reporting System (SOARS).

Generally, most large-scale engineering development projects adhere to and maintain international quality standards that mandate the storage of Quality Management System records. These records range from failure reporting and root cause analysis repositories, product warranty databases, anomaly and problem reporting systems and provide an excellent resource for records of anomalous behavior.

4.2.2 Design and Use Environment Data

We regard precedent systems as any operational or previously operational system that has completed its primary mission for which manufacturing, project management, and test and operational performance data have been documented. The comparison and characterization process accounts for the operational and environmental effects due to the differences in use between precedents and concepts. Multiple precedents can be utilized in augmenting the data for defining the spatial field of the family of systems.

4.2.3 Expert Opinion Data

An expert is an individual with a high level of skill and knowledge pertaining to the system or system attribute of interest. This individual is conversant with technologies and processes associated with the particular system or subsystem characteristic and is able to assess the impact of design decisions with respect to both the precedent and concept systems. The expert has the ability to assess alternative attributes and provide relative qualitative and quantitative measures of attribute proximity with respect to the context of comparison. Essentially, the expert provides opinion on the effects, on engineered systems, of natural selection indirectly imposed during the design process.

Opinion data is either 1) intuited, when comparing categorical or qualitative attributes such as color, gender, country, organization, or 2) based on quantitative or numerical variables, when comparing quantifiable attributes such as fuel efficiency, mass, power output, or temperature. The result of the elicitation is a set of pairwise measures of proximity, $\delta_{i,j}$, representing the psychological distance for categorical variables, and for numerical variables, $d_{i,j}$ representing the actual quantitative distance between the i^{th} and j^{th} attribute. Table below is a grouping of comparable variables as either categorical or numerical. Pairwise assessments of proximity between entities for any of the variables in Table 4.1 can be performed provided a clear context of comparison.

Table 4.1
Examples of Categorical and Numerical Variables

Categorical Variables	Numerical Variables
Gender	Fuel Efficiency
Color	Mass
Organization	Power Output
Country	Grade Point Average
Spacecraft Stabilization	Height
Spacecraft Type	Spacecraft Delta-V

4.3 Spatial Configuration from Subjective Measures of Proximity

In the preceding section, we outlined the broad aspects of a model for estimating a probabilistic metric of interest of a system based on relevant historical data. We also delineated the general types of data necessary to feed such a model while tacitly implying that a transformation of the subjective data is necessary, i.e., subjective rank-ordered proximity from expert opinion $\delta_{i,j}$ have to be transformed into quantitative measures of similarity, $D_{i,j}$.

To achieve this, we develop spatial models based on the multivariate analysis technique of nonmetric multidimensional scaling. Our resultant spatial models are geometric in that they use relative positions in an abstract space to represent objects that can interpreted as distance data [45].

As discussed in Section 3, NMDS is an ordination technique intended to reduce the degrees of dimensionality of data so that they can be spatially represented with consideration of only rank-ordered proximity, $\delta_{i,j}$. NMDS methods are designed to produce a spatial configuration that consolidates the information in a data set into a map-like graphical representation such that the axes of the map align with latent dimensions of variation in the data. The difference in our application is that we prescribe these latent dimensions as the context of comparison and then evaluate attributes of pertinence to that context.

The reliance on rank orders alone sets NMDS apart from other ordination methods such as Principal Coordinate Analysis that require Euclidean distances as input. The advantage and flexibility offered through the use of rank orders allows extension of ordination techniques to the assessment of psychological estimates of proximity regardless of the entities being compared.

NMDS methods are traditionally used to investigate the attributes in multidimensional data which most influence psychological judgments of proximity or preference by reducing the dimensions and reconstructing the spatial configuration that best preserves the rank order of the judgments with minimal stress. In our utilization of NMDS, we, *a priori*, prescribe the context (e.g. reliability), and dimensions of comparison through the hierarchical decomposition of the system as illustrated in Figure 3.2 and Appendix E.1.

Given a set of subjective pairwise proximity values for a family of objects, we first find the spatial configuration of that family of systems and calculate the interpoint distances between all members within that family.

4.3.1 Spatial Configuration of Colors

Using data from Ekman's experiment on dimensions of color vision, [46], we illustrate the process of recovering the relational map of entities which conveys proximity.

For a family of N systems, a total of $N(N-1)$ pairwise measures of proximity can obtained from a comparison of inter-family systems. If similarity is assumed to be symmetric, then these subjective pairwise measures of proximity provide $\frac{N(N-1)}{2}$ input measures from which to obtain the spatial solution of lowest stress in at most $N-1$ dimensional space.

Consider, then, the pairwise qualitative measures of similarity obtained from participants in Ekman's experiment. The color data consists of 31 individual, pairwise judgements of color similarity between 14 colors, resulting in a total of 91 similarity measures. In the original data, the higher scores indicates that the colors are more similar. The 31 individual ranks are averaged. Ekman's original similarity matrix converted to normalized dissimilarity values is shown in Table 4.2. The resultant matrix is a zero-diagonal symmetric matrix used as the input distance matrix to multidimensional scaling.

We perform a non-metric multidimensional scaling using the Stress Majorization of a Complicated Function (SMACOF) package [47] of the open source language R [48] on the dissimilarity matrix. The spatial configuration of the colors in two dimensions is obtained with a stress of 0.016, indicating a good solution has been found. From this spatial configuration, the inter-color distances or ordination distances can be computed. Figure 4.2 represents the two-dimensional ordination solution derived from the subjective measures of color proximity with each of the 14 colors labeled based on its particular wavelength in nanometers. It closely matches the traditional visualization of color arrangement, the well-known "color circle", which we have super-imposed on the configura-

Table 4.2
Color dissimiliarity based on psychological measures of proximity (Ekman 1954)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	1.4	5.8	5.8	8.2	9.4	9.3	9.6	9.8	9.3	9.1	8.8	8.7	8.4
2	1.4	0	5	5.6	7.8	9.1	9.3	9.3	9.8	9.6	9.3	8.9	8.7	8.6
3	5.8	5	0	1.9	5.3	8.3	9	9.2	9.8	9.9	9.8	9.9	9.5	9.7
4	5.8	5.6	1.9	0	4.6	7.5	9	9.1	9.8	9.9	9.9	9.9	9.8	9.6
5	8.2	7.8	5.3	4.6	0	3.9	6.9	7.4	9.3	9.8	9.8	9.9	9.8	9.9
6	9.4	9.1	8.3	7.5	3.9	0	3.8	5.5	8.6	9.2	9.8	9.8	9.8	9.9
7	9.3	9.3	9	9	6.9	3.8	0	2.7	7.8	8.6	9.5	9.8	9.8	9.9
8	9.6	9.3	9.2	9.1	7.4	5.5	2.7	0	6.7	8.1	9.6	9.7	9.8	9.8
9	9.8	9.8	9.8	9.8	9.3	8.6	7.8	6.7	0	4.2	6.3	7.3	8	7.7
10	9.3	9.6	9.9	9.9	9.8	9.2	8.6	8.1	4.2	0	2.6	5	5.9	7.2
11	9.1	9.3	9.8	9.9	9.8	9.8	9.5	9.6	6.3	2.6	0	2.4	3.8	4.5
12	8.8	8.9	9.9	9.9	9.9	9.8	9.8	9.7	7.3	5	2.4	0	1.5	3.2
13	8.7	8.7	9.5	9.8	9.8	9.8	9.8	9.8	8	5.9	3.8	1.5	0	2.4
14	8.4	8.6	9.7	9.6	9.9	9.9	9.9	9.8	7.7	7.2	4.5	3.2	2.4	0

tion solution to highlight the accuracy of the solution. This configuration, progressively and in a radial pattern, places colors adjacent to each other with respect to their proximity.

Considering that color wavelength is a metric measure which can be used to linearly organize colors along the spectrum of wavelengths, one would expect a one-dimensional spatial solution would adequately encode and replicate the subjective measures of dissimilarity. However, examination of the resulting scree plot from ordination of the color data shows a significant decrease, marked by a distinct "knee", as we move from the one-dimensional to the two-dimensional solutions. This indicates that there may be additional factors other than differences in wavelength when it comes to human perception in judging proximity of colors. Moving past the second dimension, there is little reduction in the Kruskal stress, suggesting that two dimensions are adequate for the evaluation.

We use this illustration to establish the progression of our methodology. From pairwise comparison of proximity, ordination returns the underlying spatial configuration of entites. Next we investigate if attributes of these entities can be estimated from the spatial configuration. The obvious, if trivial answer, in light of the color exercise, is yes. One can potentially interpolate between a spatial arrangement of wavelengths to estimate a wavelength at an unknown location around the color circle. For completenes, however, we demonstrate this spatial inference capability in application of the developed model in Chapter 6.

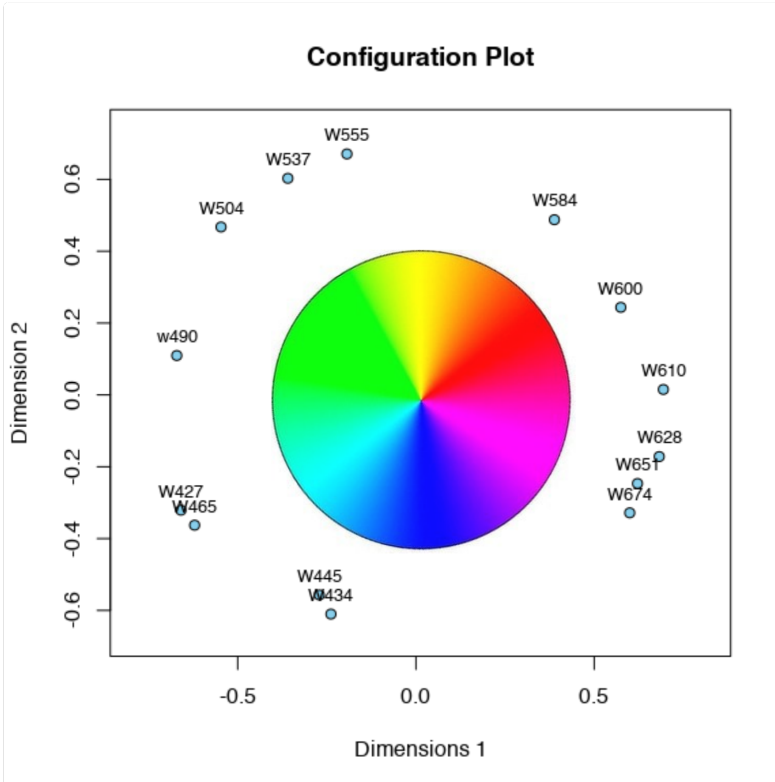


Figure 4.2

Spatial configuration of 14 Colors based on subjective dissimilarity values

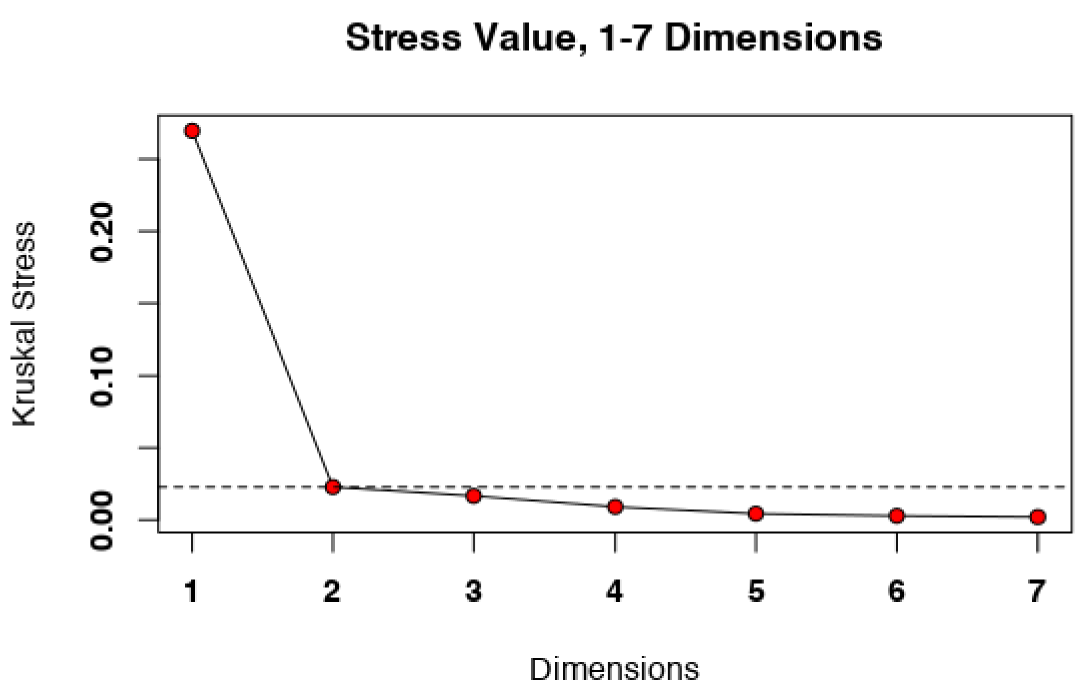


Figure 4.3

Scree plot of color dissimilarity ordination

Through multidimensional scaling, the true spatial configuration of colors with respect to their similarity has been determined based entirely on subjective ideas of color similarity. The spatial arrangement, together with the implicit coordinate locations of each color, provides a means for extracting interpoint distances between the colors. More importantly, the collection of color location and possible observations at each location fits a class of data ideal for spatial analysis.

5 Bayesian Inference Model for Pseudo-spatial Processes

Given geographically referenced data across disciplines including public health, political science, meteorology, etc., it is common practice to carry out statistical analysis tasks like modeling of trends, estimation of parameters, or prediction of outcomes at unmonitored sites. However, our research interest is whether the same inference methods apply to pseudo-spatial data when the spatial relatedness is deduced from largely intuitive ideas of proximity?

Fundamentally, the primary objective of this thesis, applicability of partially relevant data in inference tasks, is essentially a conditional probability statement and therefore tailor-made for Bayesian methods. It follows that any equivocation or uncertainty on the applicability of underlying methods can be completely handled in the Bayesian paradigm. We proceed, then, with confidence in the notion that pseudo-spatial data can be treated as being spatially referenced and explicate our Bayesian inference model for pseudo-spatially referenced data.

Spatial data sets are classified into three basic types. From [49];

1. *Point referenced data* - where $E(x)$ is a random vector at a location $x \in R^r$, x varies continuously over D, a fixed subset of R^r *fix*
2. *Area or Lattice data* - D is a fixed subset of the space R^r, however it is partitioned into irregular areas or regular lattices and the realizations of the random vector, $E(x)$, is averaged over each areal unit.
3. *Point pattern data* - the fixed study area or spatial domain, D, is itself a collection of random points. This means that not only are the realizations at point random, but the locations are also random. The location of trees in a forest, combined with the height of a tree given its location is a good example of this type of data.

Of the three types of spatial data, point-referenced data is the best suited to the data structure for our problem. We summarize our data structure as follows:

- A number of systems from a family of systems form a fixed subset, D, of the entire family-specific
- Individual designs occupy specific locations marked by coordinates, $\boldsymbol{x}$, such that identical designs coincide in the space
- System performance, $E(x_i)$ for each design x_i, are measurable observations such as records of failure

Point referenced spatial data models are underscored by a stochastic process that can be defined as;

$$\{E(x) : x \in D\} \tag{5.1}$$

where x varies continuously over D, a fixed subset of a $d-$dimensional Euclidean space for which interpoint distances, $d_{i,j}$, can be determined via Equation (3.1). The process is said to be a spatial process where $r > 1$, [49]. $E(x)$ represents an observation at location x and our data set consists of observations at finite locations $x_1, ..., x_n$ [49]. These observations are a partial realization of the stochastic process on the continuum D, and our task is to infer $E(x)$, the true stochastic process, replete with its parameters, in order to predict at new locations based on the partial realization.

5.1 Gaussian Spatial Process Models

Consider a set of spatial point referenced data, let the spatial process at a location in the field, $x \in D$ be described by the Gaussian process in Equation (3.15), modified and restated here for ease of reference.

$$\boldsymbol{E}(\boldsymbol{x}) = \mu(\boldsymbol{x})\boldsymbol{\beta} + W(\boldsymbol{x}) + \boldsymbol{\epsilon}(\boldsymbol{x}) \tag{5.2}$$

$W(\boldsymbol{x})$, as before, is a zero-mean stationary Gaussian process that accounts for spatial dependence whose variance is parameterized by $\{\sigma^2, \phi\}$, reducible to $\{\sigma, \phi\}$, if necessary.

To enable Bayesian inference we cast the point process in Equation (5.1) as a multivariate Gaussian process with a linear predictor for its mean given by Equation (5.2), such that, given a set of realizations at source locations, $\boldsymbol{Y} \equiv \{E(s_i)\}$ and coordinates $x_i, i = 1, ..., n$, the multivariate Gaussian would allow inference at target locations, $\boldsymbol{T} \equiv \{T(t_i)\}$. We note that the spatial process includes covariate data for both observed and unobserved locations and set the point process in Equation (5.1), $\boldsymbol{E}(\boldsymbol{x}) = \{\boldsymbol{Y}(s), \boldsymbol{T}(t)\}$ in Equation (3.15). Recognizing that our location vector $\boldsymbol{x}$ includes $\boldsymbol{s}$ and $\boldsymbol{t}$ we rewrite equation Equation (3.17) as:

$$\boldsymbol{E}(\boldsymbol{x})|\mu(\boldsymbol{x})\boldsymbol{\beta}, \boldsymbol{\theta} \sim MVN(\mu(\boldsymbol{x})\boldsymbol{\beta}, \Sigma(\boldsymbol{\theta})), \tag{5.3}$$

where $\mu(\boldsymbol{x})\boldsymbol{\beta}$ is the mean of the process, $\Sigma(\boldsymbol{\theta})_{ij}$ covariance between design responses $E(x_i)$ and $E(x_j)$, and $\boldsymbol{\theta}$ is the vector of parameters, σ^2, τ^2, and ϕ (for the exponential case), that defines the covariance function. The covariance is given by:

$$\Sigma(\boldsymbol{\theta}) = \sigma^2 \boldsymbol{H}(\phi) + \tau^2 \boldsymbol{I} \tag{5.4}$$

The process is Gaussian if, for any integer, $n \geq 1$ and set of locations $\{x_1, .., x_n\}$, the joint distribution of $\boldsymbol{E} = (E(x_1), .., E(x_n))^T$ is a multivariate Gaussian. Furthermore, locations, $x_1, ..., x_n$, uniquely identify the positions occupied by designs variants within a family of engineering systems in a pseudo-spatial configuration of designs.

If the mean is constant, $\mu(\boldsymbol{x}) = \mu$, as in simple kriging, then the process is **weakly** stationary, and, for any integer, $n \geq 1$, any set of locations $\{x_1, .., x_n\}$, and a separation vector, $\boldsymbol{d}$, in the $d-$dimensional Euclidean space, the covariance between observations at any pair of locations is solely a function of the inter-point distance d_{ij} (in $\boldsymbol{d}$) and underlying parameters $\boldsymbol{\theta}$ of the spatial field;

$$Cov(Y(x), Y(x + \boldsymbol{d})) = C(\boldsymbol{d}) = f(\boldsymbol{\theta}, d_{ij}) \tag{5.5}$$

For example, the exponential form for the covariance can be written as:

$$C(d_{ij}) = \sigma^2 e^{-\phi d_{ij}} \tag{5.6}$$

Parameters σ^2 and ϕ are referred to as the *partial sill* (or spatial effect variance), and the exponential decay parameter respectively. The decay parameter is used to define the range of the covariance function $r = 1/\phi$. In the case that $d_{ij} = 0$, plausible when $i = j$, a non-spatial effect variance called the *nugget*, τ^2 is included in the exponential covariance specification and together with the spatial effect variance , defines the *sill*, $\sigma^2 + \tau^2$.

The Gaussian process, Equation (5.1), has intrinsic stationarity, meaning that the expectation $E[\cdot]$;

$$E[E(x + \boldsymbol{d}) - E(x)] = 0$$

Therefore,

$$E[E(x + \boldsymbol{d} - E(x)]^2 = Var(E(x + \boldsymbol{d}) - E(x)) = 2\gamma(\boldsymbol{d}) \tag{5.7}$$

Equation (5.7) is valid when the variance depends only on the separation vector $\boldsymbol{d}$. Expanding the right side of Equation (5.7) further:

$$
\begin{aligned}
2\gamma(\boldsymbol{d}) &= Var(E(x+\boldsymbol{d}) - Y(x)) \\
&= Var(E(x+\boldsymbol{d})) + Var(E(x)) - 2Cov(E(x), E(x+\boldsymbol{d})) \\
&= C(\mathbf{0}) + C(\mathbf{0}) - 2C(\boldsymbol{d})
\end{aligned}
$$

The term, $2\gamma(\boldsymbol{d})$ is called the *variogram*. The variogram of a spatial stochastic process is given by the function:

$$
Variogram(x, x+\boldsymbol{d}) = \frac{1}{2} Var[Y(x+\boldsymbol{d}) - E(x)] \tag{5.8}
$$

It is related to the covariance function by:

$$
\gamma(\boldsymbol{d}) = C(\mathbf{0}) - C(\boldsymbol{d}),
$$

where $\gamma(\boldsymbol{d})$ is the *semivariogram*.

For an isotropic process, the semivariogram is related to the separation vector through its length $||\boldsymbol{d}||$ and any valid variogram is constrained to being a *negative definite* function. This means that for any set of locations $\boldsymbol{x}_1, ..., \boldsymbol{x}_n$, and any set of constants $a_1, ..., a_n$, such that $\sum_i a_i = 0$, if $\gamma(\boldsymbol{d})$ is valid, then

$$
\sum_i \sum_j a_i a_j \gamma(\boldsymbol{x}_i - \boldsymbol{x}_j) \leq 0 \tag{5.9}
$$

In [49], Banerjee develops the proof of this and also describes the positive definiteness condition for covariance functions.

Finally, the process is *ergodic* if $C(\boldsymbol{d}) \to 0$ as $||\boldsymbol{d}|| \to \infty$, where $||\boldsymbol{d}||$ is the length of vector $\boldsymbol{d}$. This characteristic implies that the covariance between realizations at two locations diminishes as the locations become further separated, consistent with Tobler's law and Shepard's theory of generalization.

Such Gaussian processes are typically used to model irregular, real-valued, spatial surfaces, however we extend them to pseudo-spatial surfaces created from psychological perceptions of proximity. In so doing, we investigate the appropriateness of the parameterized semivariogram for describing the distance-dependent spatial correlation function we seek.

Having enumerated the necessary properties of the Gaussian random field that represents our pseudo-spatial data, there are a number of families for valid covariance functions for spatial models, including exponential, Gaussian, and the Matern, available for use in our methodology. The Matern family include the exponential and Gaussian as special cases and transitioning between both is enabled by an additional parameter, ν, that controls the underlying smoothness of the process, and not surprisingly called the smoothness parameter. Given the generality of the Matern family, it becomes our choice of isotropic covariance functions that depend solely on distance.

We have introduced the possibility of using pseudo-spatially referenced data generated from ordination of perceptions of similarity between complex engineered systems, and the subsequent use of spatial inference methods to predict the unknown metric of interest. The problem becomes a matter of optimal spatial prediction; i.e., provided observations of a random field, $E(\boldsymbol{x} = E(x_1), ..., E(x_n))$, how do we predict the random variable E at a location x_0 where no responses have been observed based on the realizations of the Gaussian process, $\boldsymbol{E}$, from a collection of other locations in the spatial field? Our primary thesis objective is the conditional statement given in Equation (5.10) and is analogous to the question at hand.

$$E(\boldsymbol{x}_0)|\boldsymbol{E} \tag{5.10}$$

By treating the collection of observed data and associated unknown parameters of the spatial field as random variables, Bayes Theorem, Equation (5.11), provides a structure for combining evidence with subjective opinion or other information to update the state of knowledge regarding the uncertain random variables.

$$p(E(x_0) \mid E(x)) = \frac{L\{E(x) \mid E(x_0)\} \times p(E(x_0))}{\int\limits_{E(x_0)} L(E(x \mid E(x_0))\, p(E(x_0))dE(x_0)} \tag{5.11}$$

Eq.(5.11) is the Bayesian expression of the posterior distribution of an observation at an unmonitored location, x_0, given the partial realization of the stochastic Gaussian process to which it belongs.

The likelihood term in Eq.(5.11) connotes a mapping of the partially relevant information from one system to another different system and that given the true value of the parameters used in predicting the realization, the same stochastic Gaussian process would result. Given the colloquial description of Eq.(5.11), part of the task is ensuring that the data from the precedent is metered in accordance with the degree of similarity between both systems in order to accurately update the prior value of $E(\boldsymbol{x}_0)$.

Before delving into the inference problem, i.e., predicting observations $E(x_0)$ at unmonitored locations (or performance of an unbuilt system), we must first characterize the spatial process, Equation (5.1), that contains the unobserved coordinates of interest. This means that we have to estimate the posterior distribution of the underlying parameters of the family-specific spatial process.

Examining Equation (5.3), we note that it is essentially the likelihood expression for the realization of the multivariate Gaussian process. Based on this observation, we can write the complete, albeit in compact notation, form of the Bayesian expression for the joint posterior distribution of the Gaussian process:

$$p(\boldsymbol{\theta} \mid \boldsymbol{E}(\boldsymbol{x})) = \frac{L(\boldsymbol{E}(\boldsymbol{x}) \mid \boldsymbol{\theta}) \times p(\boldsymbol{\theta})}{\int\limits_{\boldsymbol{\theta}} L(\boldsymbol{E}(\boldsymbol{x}) \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}) d\boldsymbol{\theta}} \tag{5.12}$$

Eq.(5.12) is the Bayesian expression of the joint posterior distribution of a set unknown of parameters and covariates, $\boldsymbol{\theta}$, associated with the pseudo-spatial process to which both concept and precedent systems belong conditional on the partial realizations of the process.

5.2 Non-Gaussian Spatial Process Models - Implications for the Likelihood

The likelihood term in Eq.(5.12) must address the degree of applicability of the model through which the precedent failure data is generated to the concept's design or underlying failure process. In Bayesian analysis, the likelihood term conceptually represents the process through which data is generated. In our methodology, observations can be modeled as random events or, alternatively, in combination with deterministic physical phenomena. Modeling of both random and deterministic failure processes through the use of probabilistic physics of failure models combined with statistical models for random processes determine the likelihood function for the particular system under study. Our focus in this section is to develop the likelihood expressions for realizations of a spatial point process that are not necessarily Gaussian.

The foregoing is extended to non-Gaussian process where the realizations of the process manifest as either dichotomous data e.g. the presence or lack thereof of a signal, or count data, e.g. the number of anomalies over a finite period. In these cases, it is important that the underlying data generating process be accurately modeled. The general linear Gaussian process is still well-suited to handle these data.

5.2.1 Dichotomous Data

Extension of spatial process models to dichotomous data is achieved by redefining the likelihood expression in Equation (5.12) as follows:

$$\boldsymbol{E}(\boldsymbol{x}) \mid \boldsymbol{\pi} \sim Bernoulli(\boldsymbol{\pi}) \tag{5.13}$$

where π is linked to the latent spatial field parameters, $\boldsymbol{\theta}$, via the cumulative density function of a standard Gaussian distribution, $\Phi(\cdot)$, i.e.;

$$\pi = \Phi(\boldsymbol{\theta})$$

As before, $\boldsymbol{\theta}$ is a collection of parameters that characterize the latent spatial field effect.

5.2.2 Count Data

Similarly, spatial count data can be modeled by redefining the likelihood as:

$$\boldsymbol{E}(\boldsymbol{x}) \mid \boldsymbol{\lambda} \sim Poisson(\boldsymbol{\lambda}) \tag{5.14}$$

where λ is the average count of events linked to the latent spatial field parameters via a logarithm link function, i.e.;

$$log(\lambda) = \boldsymbol{\theta}$$

5.3 Gradient of Generalization - The Matern Family of Covariance Functions

In Chapter 2, we reviewed literature [13], [15] in which the probability of generalization, derived from a gradient, is defined as the conditional probability of eliciting a response from a stimulus that has been associated with a different stimulus. It has been postulated [14] that if metric measures of similarity are recovered from psychological measures of proximity [17], then owing to the invariance of psychological space, a monotonically decreasing function relates the conditional probability of generalization to the separation distance. This relationship between the conditional probability of generalization and the separation distance is the gradient of generalization.

Mathematically, the gradient must reflect a probability of generalization that approaches unity as the dissimilarity approaches zero, and a diminishing probability of generalization as dissimilarity or distance increases. In the context of our model, the gradient of generalization determines the degree to which attributes of one entity or a collection of entities can be associated with neighbors. Obviously, if entities occupy the same location in a psychological space, then they are identical and the chances of generalizing attributes from one to the other is total.

In describing a stationary, isotropic Gaussian process for point-referenced data, the covariance between any pair of points or locations in the same Gaussian field, is strictly a parameterized function of the inter-point proximity. Within the Gaussian field, the parameterized covariance between the i^{th} and j^{th} points is given in Equation (5.15). When normalized, the parameterized covariance function, ρ, of interpoint distance is synonymous with the generalization gradient captured in Shepard's Theory of Generalization.

$$(\boldsymbol{H}(\phi))_{ij} = \rho(\phi; d_{ij}), \tag{5.15}$$

Expressed as an exponential decay function, in Equation (5.6) the normalized covariance is a bounded, non-negative, and differentiable value [50] that depends on the continuous random variable $d_{i,j}$ and the underlying vector of parameters, $\boldsymbol{\theta} = \{\sigma, \phi\}$. Figure 5.1 below illustrates the dependence of the conditional probability on the underlying rate parameter, ϕ and the distance measure, $d_{i,j}$. The three curves are exponential decay functions with different rate parameters and a partial sill, σ^2, of one. Irrespective of the rate parameter, we see that the probability of generalization or the normalized covariance is 1 when the distance is zero, i.e. the designs occupy the same location in the Gaussian field therefore all of the precedent failure data, or other observations can be generalized from one design to the other with complete certainty.

Figure 5.1 is a monotonically decreasing function whose decay rate, in conjunction with the distance measure, determines the value of the probability of generalization. $\boldsymbol{\theta}$ parameterizes the gradient of generalization and can be estimated from evidence through the use of Bayesian methods, Equation 5.12.

As previously stated, the exponential function, as an option for ρ, is a particular case of the more flexible Matern family of covariance, which we adopt for modeling pseudo-spatial correlation.

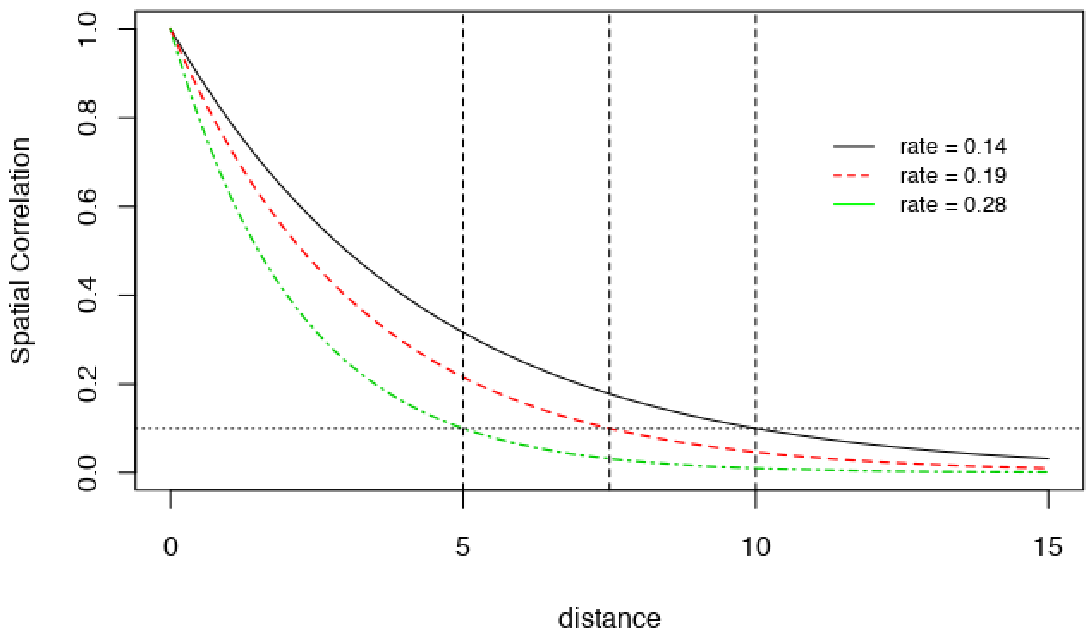


Figure 5.1

Exponential correlation function

Equation (5.16) is the Matern variogram and Equation (5.17) is the corresponding Matern covariance.

$$\gamma(\boldsymbol{d}) = \begin{cases} \tau^2 + \sigma^2 \left[1 - \frac{(2\sqrt{\nu} d\phi)}{2^{\nu-1}\Gamma(\nu)} K_\nu(2\sqrt{\nu} d\phi)\right] & \text{if } d > 0 \\ \tau^2 & \text{otherwise} \end{cases} \tag{5.16}$$

$$C(\boldsymbol{d}) = \begin{cases} \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)} (2\sqrt{\nu} d\phi)^\nu K_\nu(2\sqrt{\nu} d\phi) & \text{if } d > 0 \\ \tau^2 + \sigma^2 & \text{otherwise} \end{cases} \tag{5.17}$$

The covariance function, Equation (5.17), allows specification of the parametric form of ρ, the valid correlation function in Equation (5.15).

$$\rho(\phi; d_{ij}) = \frac{1}{2^{\nu-1}\Gamma(\nu)} (2\sqrt{\nu} d\phi)^\nu K_\nu(2\sqrt{\nu} d\phi) \tag{5.18}$$

K_ν in Equations (5.16), (5.17), and (5.18) is the modified Bessel function, $K_\nu(\cdot)$, of order ν, [51], where ν is a smoothness parameter that returns the Matern function to an exponential function

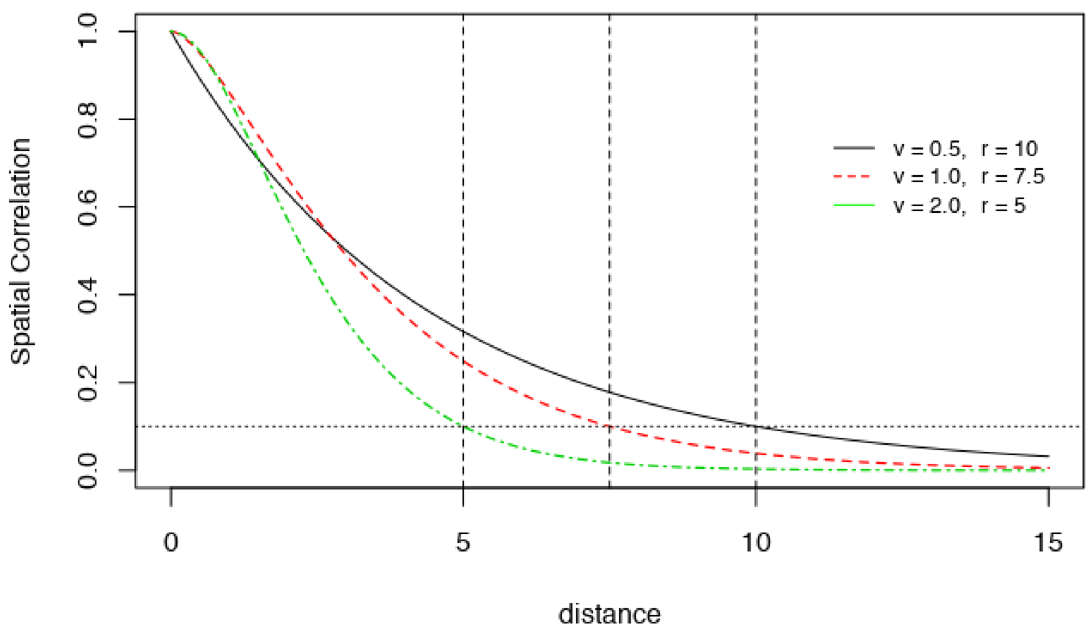


Figure 5.2

Matern correlation function

when $\nu = 0.5$.

Figure 5.2 illustrates a plot of three Matern correlation functions for different values of ν and ϕ. Recall that ϕ is the decay rate parameter that determines the range, the distance at which the probability of generalization (spatial correlation) is becomes negligible. The range is typically set as the distance at which the spatial correlation drops to 0.1 or below.

5.4 Bayesian Estimation of Spatial Field Parameters

To estimate the Gaussian process parameters, we adopt a two-stage hierarchical model. The Gaussian process, Equation (5.2), accounts for the possibility of a latent Gaussian field of random effects that have spatial correlation through the term $W(x)$. We previously defined this term as a zero-mean Gaussian with a variance of $\sigma^2 \boldsymbol{H}(\phi) + \tau^2 \boldsymbol{I}$, where $H(\phi)$ is defined in Equation (5.15) and ρ is a valid correlation function parameterized by ϕ and, $d_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$ (the Euclidean distance between the *ith* and the *jth* elements. Incorporating into Equation (5.3);

$$\boldsymbol{E}(\boldsymbol{x})|\mu(\boldsymbol{x})\boldsymbol{\beta},\boldsymbol{\theta} \sim MVN(\mu(\boldsymbol{x})\boldsymbol{\beta},\sigma^2\boldsymbol{H}(\phi)+\tau^2\boldsymbol{I}), \tag{5.19}$$

and reintroducing the Gaussian process non-spatial variance term τ^2, we get the first-stage specification of the hierarchical model;

$$\boldsymbol{E}(\boldsymbol{x})|\mu(\boldsymbol{x})\boldsymbol{\beta},\boldsymbol{W} \sim MVN(\mu(\boldsymbol{x})\boldsymbol{\beta}+\boldsymbol{W},\tau^2\boldsymbol{I}), \tag{5.20}$$

and a second-stage specification of the latent Gaussian field of spatial effects;

$$\boldsymbol{W}|\sigma^2,\phi \sim N(\boldsymbol{0},\sigma^2\boldsymbol{H}(\phi)), \tag{5.21}$$

Equations (5.20), and (5.21), combined with (5.18) can be used to redefine the Bayesian model, Equation (5.12), for estimating the parameters of the spatial process. This results in Equation (5.22), a compacted, vector form of the two-stage hierarchical Bayesian model.

$$p(\boldsymbol{\theta},\boldsymbol{W}|\boldsymbol{E}) \propto f(\boldsymbol{E}|\boldsymbol{\theta},\boldsymbol{W})p(\boldsymbol{W}|\boldsymbol{\theta})p(\boldsymbol{\theta}), \tag{5.22}$$

5.5 Bayesian Computation via Laplace Approximation

At the core of Bayesian computation is the need to, at times, evaluate mathematically intractable, multi-dimensional integrals. With the advent of high-speed computing, the use of sampling techniques such as Markov chain Monte Carlo (MCMC), Hamiltonian Monte Carlo (HMC), has eliminated most of the early roadblocks that faced Bayesian computation. However, modeling spatial fields as Gaussian processes requires operations involving the spatial covariance function Σ and its determinant. Due to the computing capability needed for matrix operations on high-dimensionality covariance matrices; the so-called "big n" problem [49], sampling algorithms are slow in effectively exploring proposal distributions required to implement Bayes.

5.5.1 Integrated Nested Laplace Approximation

Based on the classical method of Laplace Approximation, in which the integrand is approximated with a second-order Taylor-series expansion around the mode and then analytically integrated, a version, *Integrated Nested Laplace Approximations* [52] has been developed. INLA is a nested extension of classical Laplace approximations that incorporates the use of sparse matrices to the extent that factorization of high dimensional matrices required in spatial modeling is significantly reduced.

In lieu of Monte Carlo sampling methods, we adopt INLA in conjunction with the stochastic partial differential equation (SPDE) approach [53] for our Bayesian computation.

5.5.2 Stochastic Partial Differential Equations

The SPDE approach involves representing the spatial process or Gaussian Field (GF) using a discretely indexed spatial random process such as a Gaussian Markov Random Field (GMRF)[3]. See [54] for details. The approach is based on linear fractional stochastic partial differential equation, Equation (5.23), [55]:

$$(\kappa^2 - \Delta)^{\alpha/2}(\tau\xi(s) = W(s) \tag{5.23}$$

where $s \in R^d$, Δ is the Laplacian, α is a smoothness, κ is a positively defined scale parameter, τ controls variance and $W(s)$ is a Gaussian spatial process. From [55], "the exact and stationary solution to this SPDE is the stationary GF $\xi(s)$ with Matern covariance function..." The following expressions provide the link between the SPDE terms and the Matern covariance function of Equation (5.16):
Smoothness;

$$\nu = \lambda = \alpha - d/2$$

considering the two-dimensional case, $d = 2$, therefore

$$\nu = \lambda = \alpha - 1$$

Marginal variance;

$$\sigma^2 = \frac{\Gamma(\nu)}{\Gamma(\nu)(4\pi)^{d/2}\kappa^{2\nu}\tau^2}$$

Decay rate;

$$\phi = \frac{\kappa}{2\sqrt{\nu}}$$

An SPDE solution is approximated using a finite element method with a function defined on a triangulation of the fixed spatial domain. See [55, p. 196] for further details on implementation of

[3]A GMRF is a Gaussian random variable with Markov properties

the SPDE approach.

5.6 Model Performance

To enable correction of model predictions given the possibility of model uncertainty, as discussed in, Mosleh and Droguett, [11], we formulate two simple metrics for assessing model performance. Additionally, these two metrics will provide bookend mechanisms for comparing the model against other possible estimation approaches.

5.6.1 Accuracy

Consider partial realization of a Gaussian process that includes location and observation data. Using the set of process realizations, one can conceivably withhold observations from the field and then make predictions for the locations "absent" observations. By systematically removing single realizations and then obtaining predictions for them, a set of model performance data can be generated.

The result is a set of actual observations and corresponding predictions had the observations been unknown. Defining a model performance measure Ω as:

$$\Omega = 100 \times (1 - \frac{[Predicted - Actual]}{Actual})$$

For every i^{ith} prediction, we obtain a measure of model performance, Ω_i, yielding a vector of performance measures $\boldsymbol{\Omega}$. By treating $\boldsymbol{\Omega}$ as a positively-defined, continuous random variable with possible realizations $[\Omega_1, ..., \Omega_n]$ for n pairs of model predictions and actual values, we can implement a Bayesian approach to estimating its true value.

Let $\Omega \sim Lognormal(\Omega_\mu, \sigma_\Omega)$ represent the prior distribution of the performance measure and $[\Omega_1, ..., \Omega_n]$ represent evidence, E_Ω for Bayesian updating. Then joint posterior distribution of the hyperparameters μ_Ω, and σ_Ω is:

$$p(\mu_\Omega, \sigma_\Omega \mid E_\Omega) = \frac{L(E_\Omega \mid \mu_\Omega, \sigma_\Omega) p(\mu_\Omega, \sigma_\Omega)}{\int\limits_{\mu_\Omega} \int\limits_{\sigma_\Omega} L(E_\Omega \mid \mu_\Omega, \sigma_\Omega) p(\mu_\Omega, \sigma_\Omega)} \tag{5.24}$$

Equation (7.9) provides a mechanism for characterizing the model uncertainty and possibly updating predictions.

5.6.2 Error

As a corollary to the $\%Accuracy$, we define a simple measure for the error in model prediction.

$$\%Error = 100 \times \frac{[Actual - Predicted]}{Actual}$$

This measure easily conveys the deviation of the prediction from the actual demonstrated value. Being centered on zero, a standard normal distribution provides an excellent likelihood for updating this measure with evidence from the five predictions.

6 Demonstration: Estimating Color Wavelength

Although seemingly a trivial exercise to approximate the wavelength of a color based on those of adjacent colors, we exercise our methodology in this "controlled" test where the outcomes are well-documented to illustrate its utility to inference based on pseudo-spatial data. The data in this exercise, obtained from Ekman's experiment on *Dimensions of Color Vision* has been discussed in Chapter 3.

6.1 Color Data

To recap, the data is a set of pairwise subjective measures of color proximity elicited from participants in Ekman's experiment. These proximity measures yield a spatial configuration from which color location coordinates are determined. We then combine the coordinate data with associated color wavelengths and estimate wavelengths at "unobserved"[4] locations.

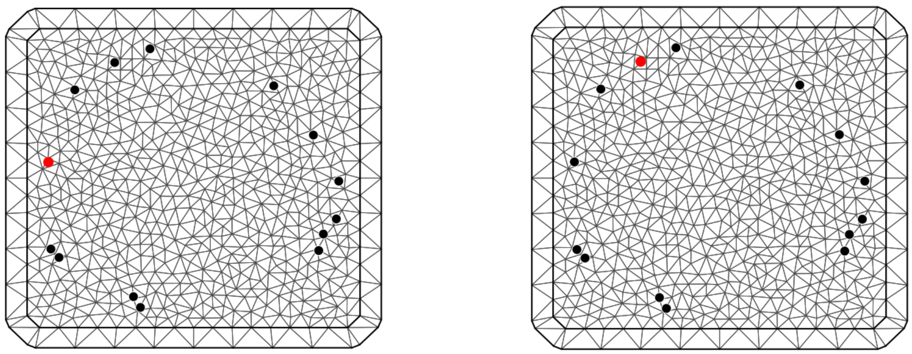
6.2 Prediction Fields

We first estimate the generalization gradient or Matern correlation function parameters for the pseudo-spatial field resulting from ordinating the opinion data. Again, note that the prediction field, contains coordinate and observation data for all colors except for the target color. This results in a set of 14 unique prediction fields, Field 1 through Field 14, with field-specific parameters for each of the predictions. Figure 6.1a is the mesh triangulation of Field 5 for estimating the wavelength of color W490, while Figure 6.1b is the equivalent field for estimating the wavelength of color W537. The red dot marks the target coordinate location, while the black dots are color locations with associated wavelengths included in the data.

6.3 Wavelength Estimation Results

Estimates for all 14 colors are provided in Table 6.1. From the table, it is apparent that estimates of the 14 color wavelengths are very close to the actual associated values, albeit with some uncertainty. This demonstrates the efficacy of the algorithm and verifies that it can provide consistently accurate estimates.

[4]at unobserved locations we withold the wavelength information for the target color



(a) Prediction Field for Color 5, 490nM

(b) Prediction Field for Color 7, 537 nM

Figure 6.1

Prediction Fields for Pseud-spatial configuration of Colors

Table 6.1

Summary of color wavelength prediction results

Color ID	Wavelength (nM)		SD	% Accuracy
	Actual	Predicted		
1	434	447	10	97.00
2	445	436	9	99.54
3	465	467	8	99.57
4	472	469	8	99.36
5	490	493	32	95.55
6	504	515	21	97.82
7	537	532	49	99.07
8	555	555	20	100.00
9	584	555	20	95.03
10	600	597	20	99.50
11	610	604	15	99.02
12	628	638	9	98.41
13	651	651	7	100.00
14	674	658	12	97.63

7 Application: Spacecraft Anomaly Prediction

We apply the framework in estimating the count of anomalies for a given mission duration for a set of US government-sponsored space science missions to illustrate its utility in infererring probabilistic measures of engineered systems. The plan for validation was to select a “concept” from a family of spacecraft for which a record of on-orbit data such as anomalies and failures have been documented. Within the family of systems, the “concept” is regarded as the system still in development but comparable to other members of the family. Then agnostic of the demonstrated anomaly rate for the “concept” we estimate its anomaly rate using operational data collected on the other in-family spacecraft. Finally, we compare our estimates to the demonstrated anomaly rate for the “concept”. Again, the “concept” system in this illustration is an operational spacecraft however we perform our comparison with its precedents based on the early design-phase level information.

7.1 Data Collection

The methodology requires three types of information; record of anomalous behavior for the precedent systems, design, development and use environment information, and expert opinion for comparison. In total, we collected data on 11 individual spacecraft.

7.1.1 Spacecraft Data

7.1.1.1 Design Information The family of spacecraft selected for comparison consisted of the nine previous space missions designed and integrated, inclusive of the concept, by a U.S-based, internationally recognized, space mission integrator over a period of 25 years.

Information regarding the design, development, testing, and use of each spacecraft was collected from a variety of sources. These include NASA mission websites and other public curators of spacecraft design and mission data. However, the most pertinent source of design knowledge came from the experts, whose familiarity with the family of spacecraft ensured an understanding of impacts of design differences.

Of the nine individual spacecraft, there are two pair of identical designs; Spacecraft G and Spacecraft H serve in multi-spacecraft missions, however unique records are still maintained for each individual. We identify them as:

- Spacecraft G.A and Spacecraft G.B
- Spacecraft H.A and Spacecraft H.B

Another significant difference between the space missions is that one of the nine, Spacecraft F, is a deep space, transit mission while the rest are in orbit around various planetary bodies.

7.1.1.2 Anomalies The data for this case study is sourced from the Anomaly, Problem, and Failure Reporting (APFR) database of the spacecraft developer. A redacted version of the data is included in the Appendices. The database is used to maintain records of anomalies and problems for each spacecraft starting from design, manufacturing, integration and test, through launch, commissioning and on-orbit operations. For the analysis, the anomaly data used is limited to all spacecraft anomalies recorded post-commissioning and attributed to spacecraft systems, not instruments or payloads, for each of the missions.

Records of anomalies for operational spacecraft are typically maintained by the organization. However, of the nine spacecraft missions considered, no performance records are available in the current reporting system on four, Spacecraft A, Spacecraft B, Spacecraft C, and Spacecraft D. This is due to the fact that three of the missions are totally operated by the sponsor organization rather than the spacecraft developer, and as a result on-orbit issues are maintained at the sponsor site. The fourth spacecraft experienced a catastrophic failure very early in its mission life and no on-orbit performance data was collected.

Two of the remaining five missions are designed with a pair of identical spacecraft (Spacecraft G.A, Spacecraft G.B, and Spacecraft H.A, Spacecraft H.B) maintaining similar orbits but separated in time. Anomaly records were maintained for each individual spacecraft.

7.1.1.3 Mission Duration Mission duration is available for all the space mission considered in this case study. The launch date, adjusted for time to orbital insertion, up till the present date was used as the effective duration for each system.

Although launched at the same time the duration data for Spacecraft H.A, and Spacecraft H.B slightly different. Communication with Spacecraft H.B was lost for a duration of 690 mission days. The significance of this is discussed further in the Results Section.

7.1.2 Expert Pool

Thirty-three experts, all employees of the integration organization, were invited to participate in the expert elicitation process. Of the 33, invitations, there were 13 respondents. Three of the 13 respondents recused themselves from pairwise comparisons which they felt they had limited experience to opine on. Results of the elicitation are in Appendix B. The expert data shows the respondents and the recusal instances. To ensure diversity of opinion, the pool of invitees consisted of the following roles from the project teams:

- Program Management
- Principal Investigators
- Mission Systems Engineers
- Spacecraft Systems Engineers
- Spacecraft Propulsion Engineers
- Guidance Navigation and Control Lead Engineers
- Electrical Power Systems Lead Engineers
- Mechanical Systems Lead Engineers
- Organization Executive Leadership
- Spacecraft Integration and Test Leads
- Mission Assurance Managers
- Spacecraft Integration Technicians

The selection of the invitees was based on two criteria; 1) the invitee had been employed by the organization for the period during which the spacecraft were developed and 2) the invitee participated in the development projects in an expert or lead role.

7.1.3 Expert Elicitation Process

To develop the pseudo-spatial configuration of the 9 spacecraft designs, we first elicited subjective measures of proximity from experts in the spacecraft design and development teams. The expert opinion elicitation process was governed by ground rules established to ensure consistency and to minimize bias. The elicited proximity values represent intuitive yet subjective views of similarity that were transformed to quantitative measures of similarity and dissimilarity for the purpose of data analysis. Respondents were asked to rank the similarity between each pair of spacecraft based on the scales of values for similarity in given in Table 7.1.

Additionally, they were asked to consider the 8 minimum factors as part of their evaluations.

Table 7.1
Scale of Values for Similarity

Value	Similarity Implication
1	Absolutely Different
2	Different
3	Different w/minor applicability of lower level elements
4	Different w/significant applicability of lower level elements
5	Interchangeable design solution
6	Interchangeable; few identical elements
7	Interchangeable; many identical elements and implementation schemes
8	Interchangeable; many identical elements, components and implementation schemes
9	Identical

These minimum factors, Table 7.2, represent a qualitative aggregation of all 42 contributory factors at the fourth level of the system hierarchy. Table 7.3 is one expert's responses on the pairwise comparison. The full set of attributes, including all 42 at the lowest level of the hierarchy, are shown in Appendix E.1.

Table 7.2
Minimum factors for pairwise comparison of spacecraft

1. Institutional philosophy on design
2. Institutional philosophy on integration and testing
3. Quality management and reliability approach
4. State of maturity of technologies used in the design
5. Effectiveness of adopted failure mitigating approaches
6. Relevant technical expertise of the project team
7. Expertise of leadership and management
8. Environmental factors and overall mission design

Table 7.3
Pairwise comparison of 9 spacecraft from Expert 33

Spacecraft	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
A	9.00	7.00	7.00	6.00	4.00	5.00	3.00	5.00	6.00
B	7.00	9.00	8.00	7.00	5.00	6.00	4.00	6.00	7.00
C	7.00	8.00	9.00	7.00	5.00	6.00	4.00	6.00	7.00
D	6.00	7.00	7.00	9.00	6.00	8.00	5.00	7.00	8.00
E	4.00	5.00	5.00	6.00	9.00	6.00	5.00	8.00	7.00
F	5.00	6.00	6.00	8.00	6.00	9.00	5.00	7.00	8.00
G	3.00	4.00	4.00	5.00	5.00	5.00	9.00	6.00	6.00
H	5.00	6.00	6.00	7.00	8.00	7.00	6.00	9.00	7.00
I	6.00	7.00	7.00	8.00	7.00	8.00	6.00	7.00	9.00

With the 36 pairwise comparisons of the 9 spacecraft within the family of systems, we determined the average ranking for each pair from the rankings of all the respondents. We then used the average similarity, Table 7.4 value for each pair as the single subjective measure of proximity.

For implementation of non-metric multidimensional scaling, we converted the average prox-

Table 7.4
Average similarity rankings from all responding experts

Spacecraft	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
A	9.00	4.48	5.14	4.36	3.15	2.95	4.24	4.62	3.98
B	4.48	9.00	5.28	3.98	4.12	3.43	3.68	4.00	4.82
C	5.14	5.28	9.00	5.61	4.82	4.09	3.31	4.24	4.81
D	4.36	3.98	5.61	9.00	5.22	5.76	3.83	4.67	5.38
E	3.15	4.12	4.82	5.22	9.00	5.44	3.46	4.94	4.24
F	2.95	3.43	4.09	5.76	5.44	9.00	3.49	4.47	4.13
G	4.24	3.68	3.31	3.83	3.46	3.49	9.00	5.15	4.14
H	4.62	4.00	4.24	4.67	4.94	4.47	5.15	9.00	4.43
I	3.98	4.82	4.81	5.38	4.24	4.13	4.14	4.43	9.00

imity measures into normalized dissimilarities as follows. First for our rank-ordered proximity measures are converted to dissimilarities by inverting each;

$$Dissimilarity, \delta'_{ij} = \frac{1}{ProximityMeasure}$$

We then normalize δ'_{ij} to a range $[0, 1]$ by rescaling as follows;

$$\delta_{ij} = \frac{\delta'_{ij} - \min(\delta'_{ij})}{\max(\delta'_{ij}) - \min(\delta'_{ij})}$$

Table 7.5 shows the resultant normalized dissimilarities elicited from our experts. This matrix, generated from the rank-ordered average proximty measures from all respondents, is the key input to the ordination process.

Table 7.5
Matrix of normalized subjective measures of dissimilarity, $\delta_{i,j}$ among spacecraft

Spacecraft	A	B	C	D	E	F	G	H	1
A	0.000	0.492	0.368	0.519	0.907	1.000	0.548	0.463	0.615
B	0.492	0.000	0.344	0.616	0.580	0.792	0.705	0.611	0.424
C	0.368	0.344	0.000	0.295	0.424	0.587	0.841	0.548	0.426
D	0.519	0.616	0.295	0.000	0.354	0.274	0.659	0.454	0.330
E	0.907	0.580	0.424	0.354	0.000	0.320	0.782	0.402	0.548
F	1.000	0.792	0.587	0.274	0.320	0.000	0.771	0.494	0.577
G	0.548	0.705	0.841	0.659	0.782	0.771	0.000	0.365	0.574
H	0.463	0.611	0.548	0.454	0.402	0.494	0.365	0.000	0.504
I	0.615	0.424	0.426	0.330	0.548	0.577	0.574	0.504	0.000

7.2 Pseudo-spatial Configuration of Spacecraft Designs

With the normalized dissimilarities in Table 7.5 serving as our input distance matrix we performed nonmetric multidimensional scaling using the Stress Majorization of a Complicated Function (SMACOF)[56] method. The transformation of dissimilarities in the input distance matrix into the configuration distances preserves the rank order of the dissimilarities which are initially recorded on an ordinal scale. The resulting configuration of spacecraft designs is given in Figure 7.1. We regard this perceptual map as a pseudo-spatial configuration of entities with quantifiable Euclidean separation distances, location coordinates, and location-specific measurements or observations (in the form of demonstrated anomalies) that can collectively be described by a spatial Gaussian process.

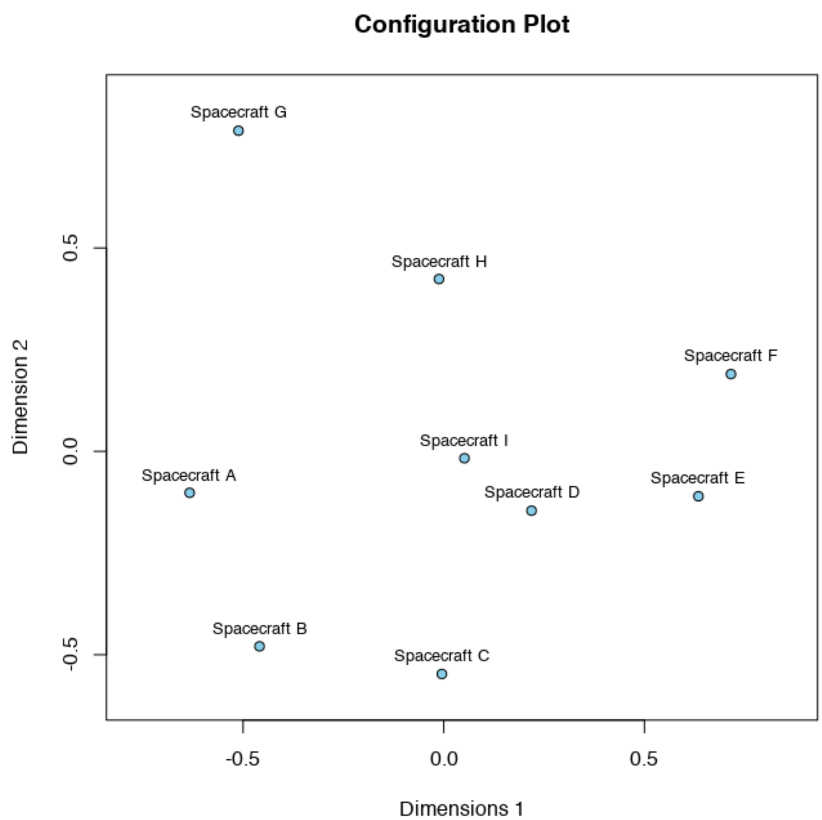


Figure 7.1

Spacecraft Configuration resulting from NMDS

The NMDS provides a number of different output. As we previously discussed, coordinates along each dimension of the acceptable solution is available in the output. In this particular case we accept the 3-dimensional solution due to the fact that it results in a Kruskal Stress value of 0.05622501, just above Kruskal's recommendation of 0.05 for a good fit. This is shown in the Scree Plot. Figure 7.2.

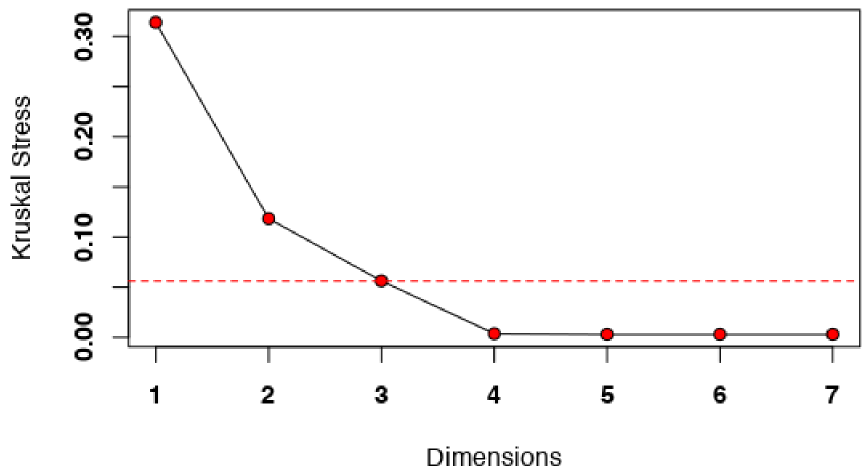


Figure 7.2
Scree Plot

From Figure 7.1, it is apparent that each spacecraft occupies a location space in the pseudo-space, with the exception of the set of identical pairs, Spacecraft H and Spacecraft G. These pairs have the same coordinates and therefore coincide in space. In Figure 7.1, we show the 2 dimensional representation of the space. Coordinates along Dimension 3 are used as altitude markers in the pseudo-spatial analysis.

The ordination solution, determined through NMDS, results in the ordination distances listed in Table 7.6. We treat these distances as quantitative measures of similarity between spacecraft with respect to the probabilistic measure of interest.

Table 7.6
Spacecraft coordinates in pseudo-spatial solution

	X	Y	Z
Spacecraft A	-0.63	-0.10	-0.33
Spacecraft B	-0.46	-0.48	0.36
Spacecraft C	0.00	-0.55	-0.11
Spacecraft D	0.22	-0.15	-0.35
Spacecraft E	0.63	-0.11	0.16
Spacecraft F	0.72	0.19	-0.28
Spacecraft G	-0.51	0.79	0.18
Spacecraft H	-0.01	0.42	-0.10
Spacecraft I	0.05	-0.02	0.47

7.3 Bayesian Parameter Estimation

Resulting from the ordination of subjective proximity measures are a set of coordinates that locate specific spacecraft designs in the collective psychological space of experts as seen in Figure 7.5. Recall that our primary objective is prediction of a response at an unobserved location given a set of locations and location-specific observations. However, to enable inference, we must first estimate the parameters of the pseudo-spatial process represented by our spacecraft 2-dimensional configuration. Equation (5.22) is the two-stage hierarchical Bayesian expression for estimating these parameters.

Equation (5.1) denotes the pseudo-spatial process associated with the occurrence of anomalies at point locations in the spacecraft spatial configuration. Since the observations in this particular example represent counts of anomalies, the parameter of interest is the average number of anomalies for over the duration of the mission. We define the distribution of the observations as:

$$y_i \sim Poisson(\lambda_i)$$

where the Laplace approximation of the Poisson distribution is performed via INLA and λ_i is related to a linear predictor, η_i via a logarithm link function:

$$\eta_i = log(\lambda_i) = \beta_0 + \sum_{m=1}^{M} \beta_m x_{im} + W(x_i) + \epsilon(x_i)$$

where $\boldsymbol{\beta}$ is the vector of regression coefficients and x_{im} is the value of the m^{th} covariate for the i^{th} spacecraft. The covariates or response predictors in our Poisson regression model are the location coordinates which characterize the pseudo-spatial field, and the mission duration.

For the case study, we specify the single covariate "Duration" and the spatial field effect. Again, the target of our Bayesian regression effort is estimating the regression coefficients and the parameters of the pseudo spatial field to enable inference of response at unobserved coordinate locations.

$$\eta_i = \beta_0 + \beta_1 \cdot Duration + W(x_i) + \epsilon(x_i)$$

Finally we select a Matern correlation function for $\rho(\boldsymbol{H}(\phi))$ for the covariance matrix of $W(x_i)$, and then group the collection of model parameters and hyper-parameters in the vector,

$$\boldsymbol{\theta} = \{\boldsymbol{\beta}, \tau^2, \sigma^2, \phi\}$$

and choose independent priors for $p(\boldsymbol{\theta})$ such that

$$p(\boldsymbol{\theta}) = p(\boldsymbol{\beta})p(\sigma^2)p(\tau^2)p(\phi)$$

thus completing the model specification. Figure 7.3 is the graphical representation of the model for Bayesian estimation of the parameters.

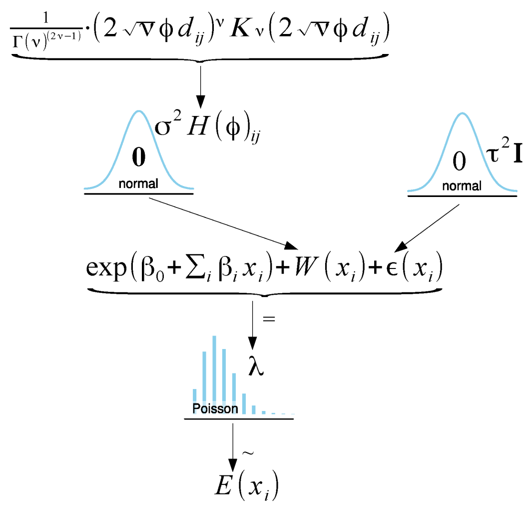


Figure 7.3

Model for estimation of pseudo-spatial field paramters

We choose vague priors for the regression coefficients $\boldsymbol{\beta}$ and the pseudo-spatial field parameter. Specification of the priors for the parameters and hyper parameters is done within the R-INLA environment using the parameterization given in [55] and the expressions relating the SPDE terms to the Matern parameters. Equations (7.1) and Equation (7.2) give the prior distributions used in the Bayesian estimation, while the anomaly data set (the realization of the pseudo-spatial process) is provided as Appendix D.

By setting the internal R-INLA paramaters, θ_1 and θ_2, to $Normal(0,1)$, the folowing priors are derived for the SPDE parameters, κ and τ, and the set of relationships between these parameters and the Matern correlation function are provided in Section 5.5.2;

$$\kappa_{prior} \sim Lognormal(-0.58, 1.01) \tag{7.1}$$

$$\tau_{prior} \sim Lognormal(-0.68, 1.42) \tag{7.2}$$

We subsequently estimate the posterior marginal distributions of each parameter using Integrated Nested Laplace Approximation (INLA) of stochastic partial differential equations (SPDE) using the R script in Appendix C.2.

Owing to the nature of the data, there are several different partial realizations of the process. This is due to the fact that the parameters of the pseudo-spatial field can be estimated by any combination of the spacecraft. We focus the parameter estimation on the pseudo-spatial field for five specific configurations; for each configuration, we hold one particular design as the target prediction location while the remainders are the source locations where observations are made. This allows post-inference comparison of the predicted response against the actuals recorded for the target in each field.

Table 7.7 lists the summary statistics of the marginal posterior distributions of the parameters and the regression coefficients of the model, while Figure 7.4 presents visualization of the posterior probability distributions. The five subtables and subplots, Table 7.7a, through Table 7.7e, and Figure 7.4a, through Figure 7.4e, respectively, represent the summaries for each of the five variations of the field. In each variation, anomaly information is withheld from the model, effectively altering the partial realization of the pseudo-spatial process.

Table 7.7
Posterior estimates (mean, standard deviation and quantiles for spatial parameters)

(a) Posterior parameters of Field 1

Parameter	mean	sd	2.50%	97.50%
τ	0.21	1.71	0.07	0.56
κ	0.79	1.97	0.21	3.02
r	3.57	1.97	0.94	13.50
σ^2	2.39	3.26	0.24	25.21
β_0	1.00	8.33	-17.49	18.95
β_1	0.0008	0.0003	0.0003	0.0014

(b) Posterior parameters of Field 2

Parameter	mean	sd	2.50%	97.50%
τ	0.32	1.88	0.08	0.97
κ	0.71	2.11	0.17	3.20
r	3.98	2.11	0.88	16.52
σ^2	1.31	3.64	0.11	17.24
β_0	1.32	7.84	-15.99	18.14
β_1	0.0007	0.0002	0.0003	0.0013

(c) Posterior parameters of Field 3

Parameter	mean	sd	2.50%	97.50%
τ	0.21	1.65	0.07	0.53
κ	0.78	1.95	0.21	2.90
r	3.65	1.95	0.97	13.44
σ^2	2.52	3.19	0.26	25.18
β_0	0.70	8.54	-18.25	19.16
β_1	0.0008	0.0003	0.0003	0.0014

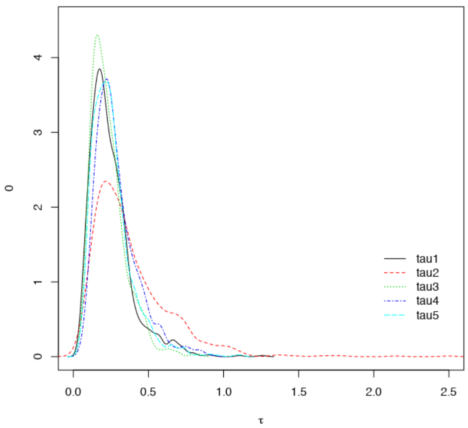
(d) Posterior parameters of Field 4

Parameter	mean	sd	2.50%	97.50%
τ	0.25	1.65	0.09	0.61
κ	0.69	1.90	0.19	2.40
r	4.13	1.90	1.19	14.81
σ^2	1.97	3.09	0.23	19.26
β_0	2.61	8.39	-16.05	20.34
β_1	0.0004	0.0004	-0.0004	0.0012

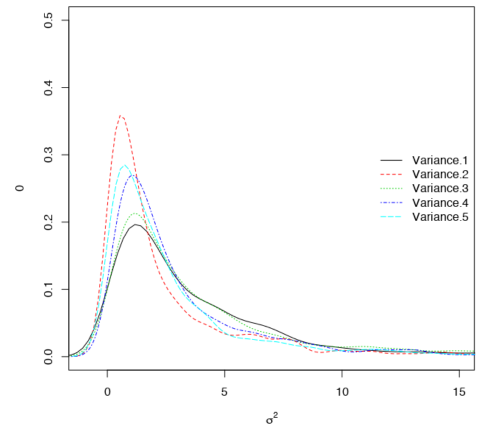
(e) Posterior parameters of Field 5

Parameter	mean	sd	2.50%	97.50%
τ	0.23	1.63	0.08	0.56
κ	0.89	2.07	0.22	3.78
r	3.18	2.07	0.75	12.99
σ^2	1.64	3.43	0.14	18.25
β_0	0.73	7.46	-15.45	16.57
β_1	0.0009	0.0003	0.0004	0.0015

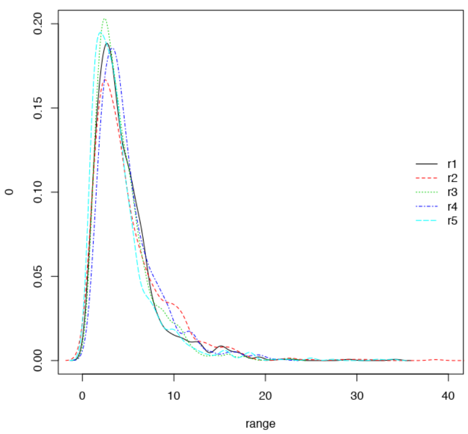
Posterior marginals of parameters of each field variation are extremely consistent with each other indicating that a fairly stable field has been established for the family of spacecraft. This observation is consistent with the fact that the Kruskal Stress of the ordination solution is within the acceptable range of ≤ 0.05. The largest discrepancy in the posterior marginals is observed in the τ parameter; recall that τ, the *nugget*, is a measure of non-spatial effect variance which is included in the Matern covariance to capture measurement error. Notice the minimal dispersion of the spatial effect variance parameter, σ in Figure 7.4b.



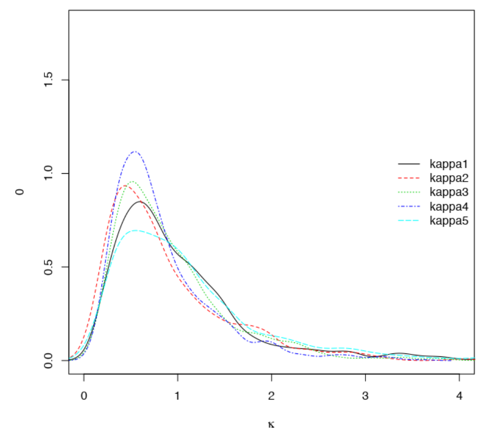
(a) Posterior marginal distributions of tau



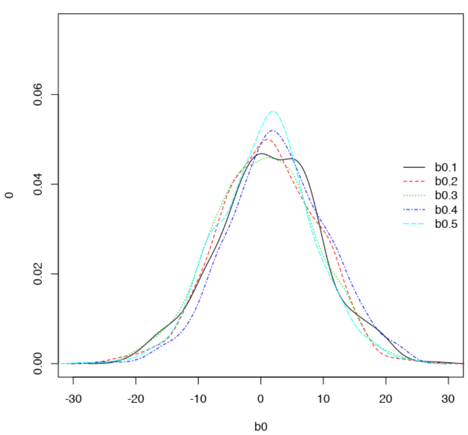
(b) Posterior marginal distributions of sigma



(c) Posterior marginal distributions of the range



(d) Posterior marginal distributions of kappa



(e) Posterior marginal distributions of b0

Figure 7.4

Posterior marginal distributions of model parameters for all five variations of the pseudo-spatial field

7.4 Anomaly Prediction

To proceed with prediction of the response, i.e. the number of anomalies for a given design, we note that the spatial field parameters were estimated without the actual observations at the validation (concept design) location. In the input data set, the concept anomaly count is withheld so as to not influence the prediction.

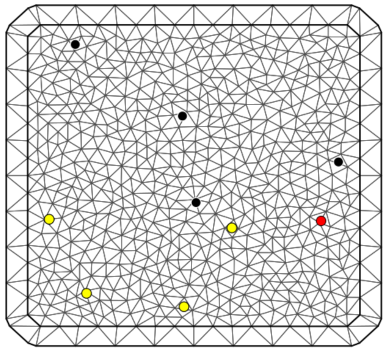
With the intent of generating predictions for each spacecraft that could be compared with actual anomaly counts, the prediction process was repeated five times using the five variations of the pseudo-spatial field, for a total of six predictions. We run five predictions since Spacecraft G.A and G.B, and Spacecraft H.A and H.B pairs of identical designs.

7.4.1 Prediction Locations

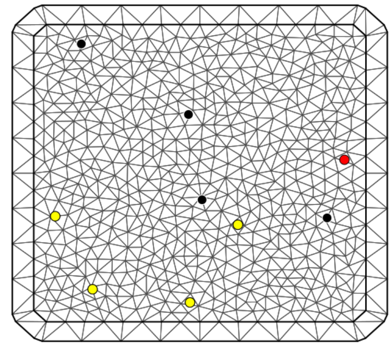
For each of the five prediction runs, we selected a particular spacecraft (or identical pair in the case G and H) as the validation or concept design and withheld its response from the data set. The locations and duration of the four missions that have no anomaly records are also maintained in the spatial process since their locations are part of the fixed study area. However, this has no effect on the inference of the spatial process parameters because every instance of the process additionally requires the associated observations at that location.

Figure 7.5 is the visualization of the R-INLA mesh triangulation of the five field variations. We distinguish the prediction target locations from the source locations as follows:

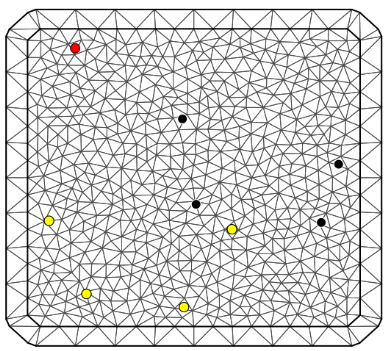
- the black dots are the source locations used to characterize the parameters of the field
- the red dot in each field is the prediction location, where anomaly/response information has been withheld
- the yellow dots represent the locations of spacecraft with no anomaly records



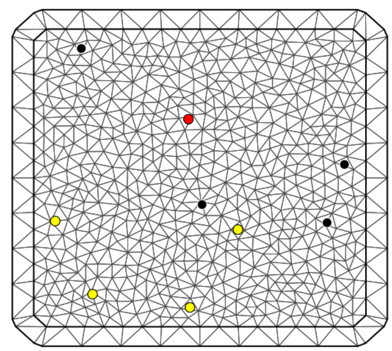
(a) Field 1 - Spacecraft E



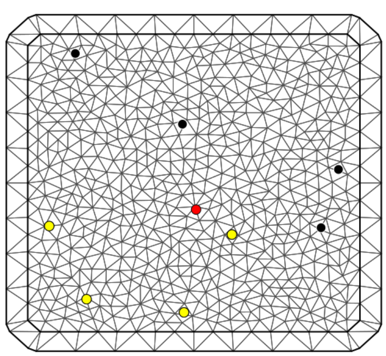
(b) Field 2 - Spacecraft F



(c) Field 3 - Spacecraft G.A and G.B



(d) Field 4 - Spacecraft H.A and H.B



(e) Field 5 - Spacecraft I

Figure 7.5

Mesh prediction fields for all five variations of the pseudo-spatial field

7.4.2 Sensitivity

We tested the sensitivity of the model to a range of conditions. By adjusting the priors on the parameters of the spatial field while leaving the dissimilarity matrix (input to the ordination process) constant. As expected, changes to the posterior marginal distributions of the field parameters and the expectations of anomaly are observed. This is typical in Bayesian analysis, where the reassignment of crediblity is dictated by the strength of both the evidence and the prior. In this example, however, our focus is on determining if subjectively created spatial fields can lend to location-specific inference. We leave as future work, the study of the implicit relationship between the input to ordination and the parameters of a spatial solution.

Testing the sensitivity of the predictions to the dissimilarity input, we note an influence on the mean predicted anomalies. These changes first manifest as alternative spatial solutions from the ordination of opinion data. The variability of the result is expected given that a totaly new Gaussian spatial process would result from assigning observations to the coordinates of this new pseudo-space. Consequently, we focus the rest of the assesment on investigating this sensitivity to uncover any attendant implications.

We anticipate that increasing the number of complete instances of the process, i.e. locations together with observations, will significantly improve the model's performance. Essentially adding more precedent spacecraft together with their anomaly records will better populate the field and increase the accuracy of the kriging interpolation.

Recall that the distance matrix is an aggregation of the subjective opinions of several experts. We subdivide the responding experts based on area of expertise and use one subgroup's aggregated dissimilarity scores to create other spatial configurations and then perform further pseudo-spatial inference.

Of our pool of experts, Mission System Engineers and Spacecraft Systems Engineers are the most conversant with all aspects of a a spacecraft by virtue of their role. These engineers are conversant with all spacecraft systems in contrast with subject matter experts such as subsystem lead engineers. With this in mind, we filter the expert opinion data and limit the input to responses from only the Systems Engineers in our pool, forming a sub-group of 10. This effectively provides us with alternative spatial solutions with which to test the model's sensitivity.

Repeating the data preparation process, we recreate the pseudo-spatial configuration based on this Subgroup. Figure 7.7 shows the resultant configuration from ordination of the Subgroup opinion data. Figure 7.8 is the Scree Plot from ordination of the sub-group opinion data. The

3-dimensional solution lies just outside the range of Kruskal's recommendation for a "Good" fit with a Stress value of 0.061. As a result we select the 4-dimensional solution, again setting the coordinates along Dimension 3 as altitude markers, and discarding Dimension 4 as signal noise. It is apparent in Figure 7.6, the ordination plot for all seven dimensions, that the configuration solution becomes relatively stable beyond the fourth dimension. This indicates that the Kruskal Stress can no longer be significantly reduced by adding dimensions.

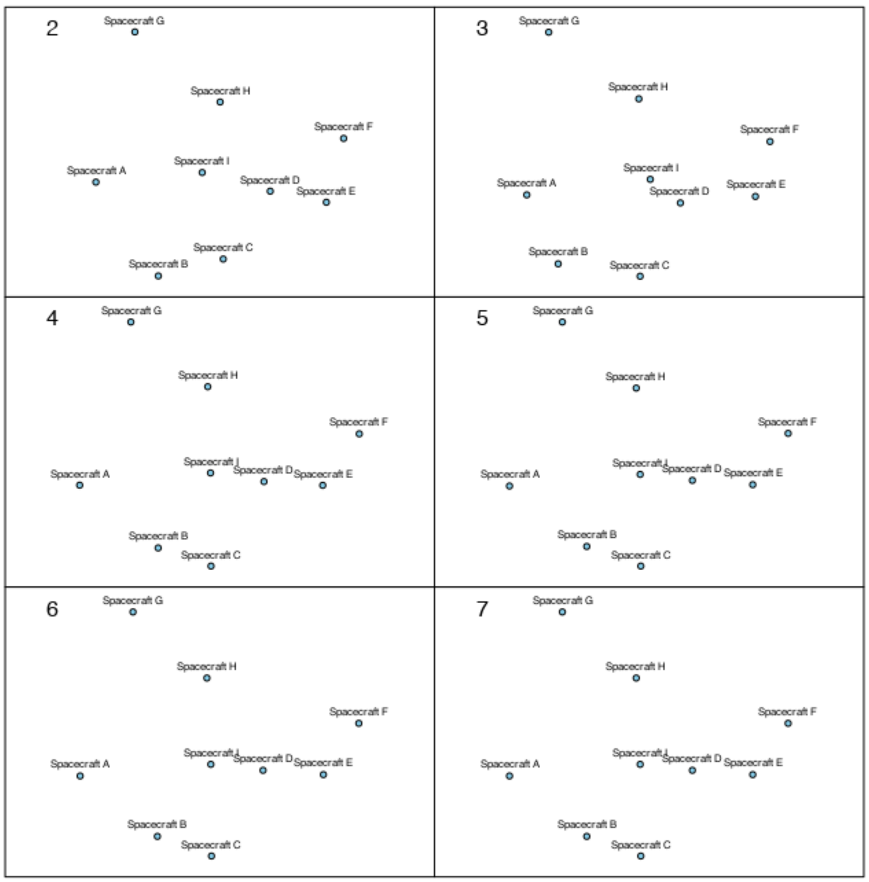


Figure 7.6

Spacecraft configuration resulting from NMDS using all expert opinion

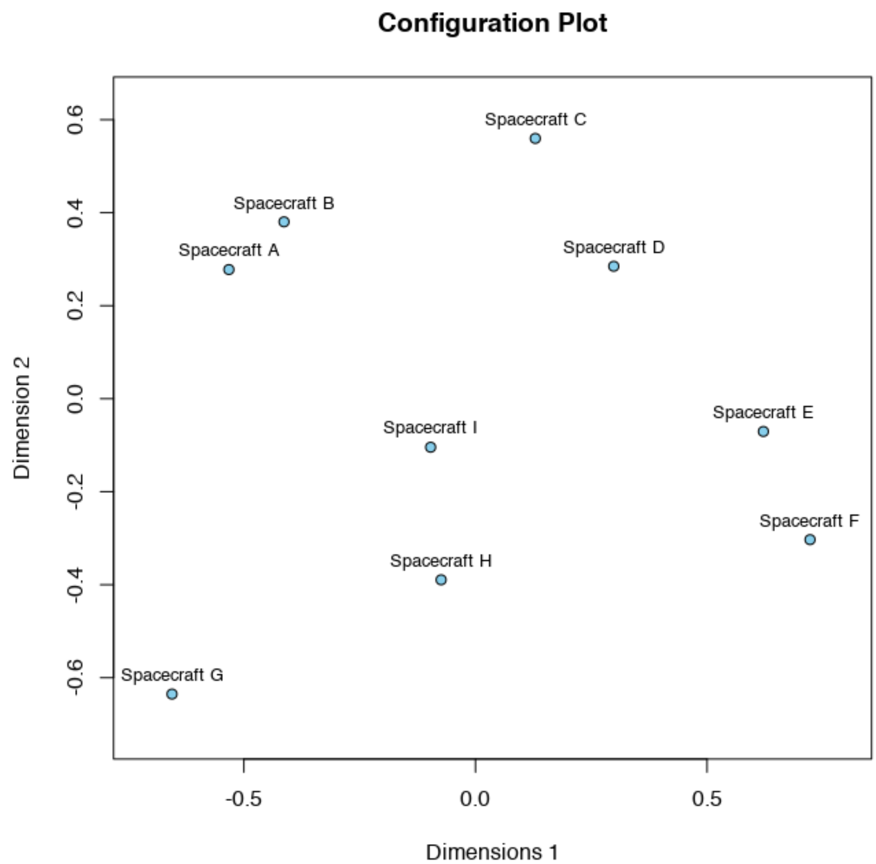


Figure 7.7

Spacecraft configuration resulting from NMDS using sub-group expert opinion

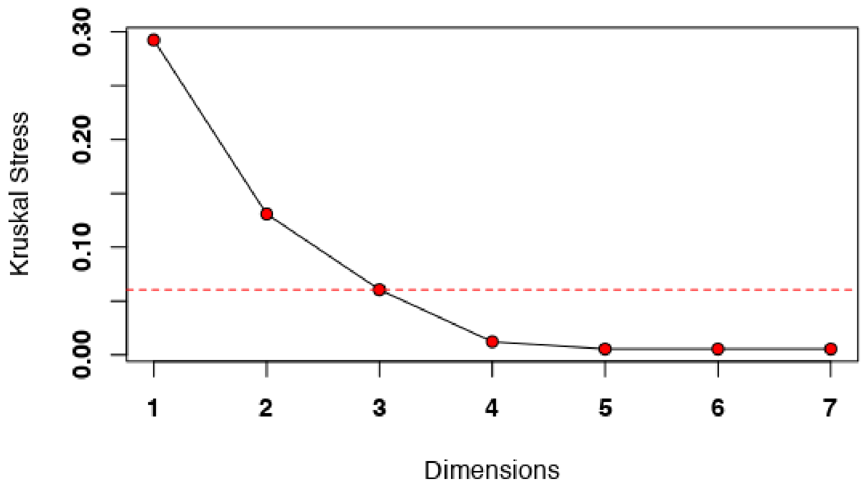


Figure 7.8

Scree Plot: Sub-group Ordination

7.4.3 Model Prediction Accuracy

To measure the model's accuracy, we derive a simple expression for percent accuracy. This allows us to compare improvements in prediction as we introduce changes in the spatial field.

$$\Omega = \%Accuracy = 100 \times (1 - [\frac{|Predicted - Actual|}{Actual}]) \tag{7.3}$$

7.4.4 Model Performance Measure Updating

In the previous chapter, we introduced a Bayesian process for describing the uncertainty in the model's performance. We proposed using a comparison between actual anomalies and predicted anomalies evidence for updating a vague prior on the performance measure. In this spacecraft example, we define the performance measure Ω by Equation (7.3).

Figure 7.9 depicts the directed acyclic graph for the Bayesian updating of the model performance measure given the five instances.

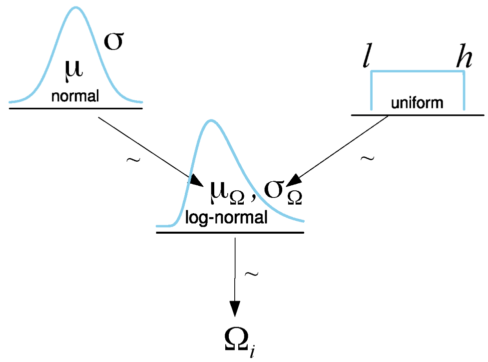


Figure 7.9

Bayeian DAG for performance measure updating

The posterior distribution of the performance measure parameters given the evidence, $\mathbf{\Omega} = \{\Omega_i : i = 1, .., 5\}$ is:

$$p(\mu_\Omega, \sigma_\Omega|\mathbf{\Omega}) \propto L(\mathbf{\Omega}|\mu_\Omega, \sigma_\Omega) \times p(\mu_\Omega, \sigma_\Omega) \tag{7.4}$$

We specify a Lognormal likelihood on the performance measure with a logarithmic mean, μ_Ω, and a logarithmic standard deviation, σ_Ω. Under assumption of independence between the parameters, we choose a diffuse normal prior for the parameter, μ_Ω with hyperparameters μ, σ, and wide uniform prior on σ_Ω with hyperparameters l, h.

Figure 7.10 and Figure 7.11 show histograms of MCMC samples generated from simulations of the posterior distributions of the mode, mean, and standard deviation of the performance measure, while Figure 7.12. The JAGS code for the model is attached as an Appendix.

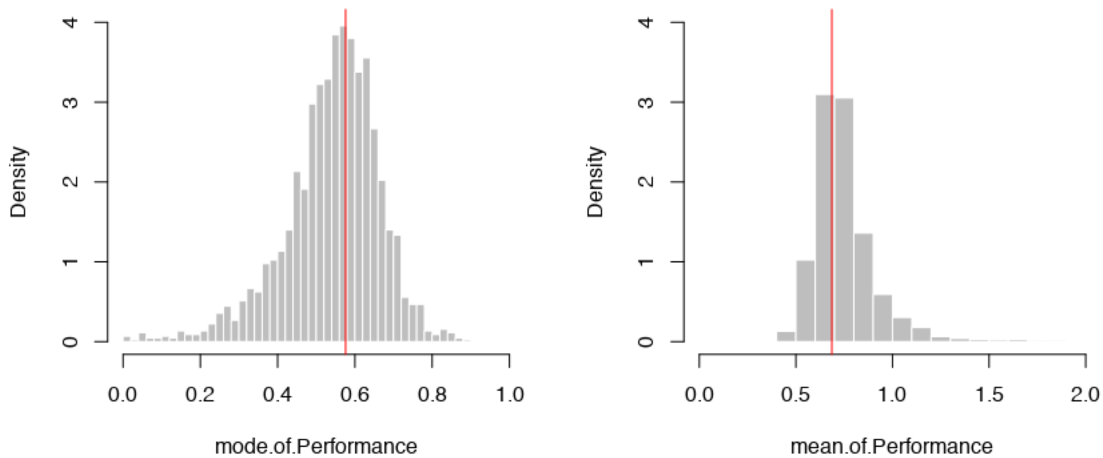


Figure 7.10

Histogram of posterior mode and mean distribution samples

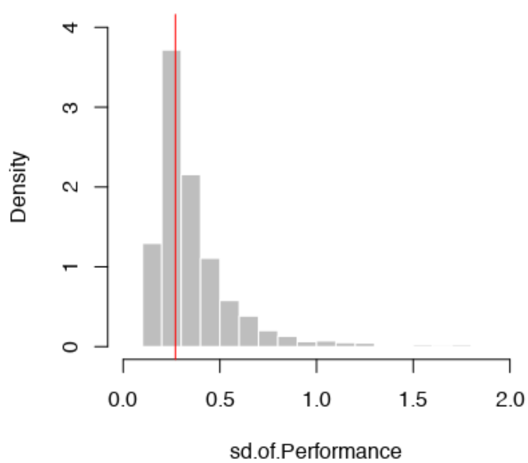


Figure 7.11

Histogram of posterior standard deviation distribution samples

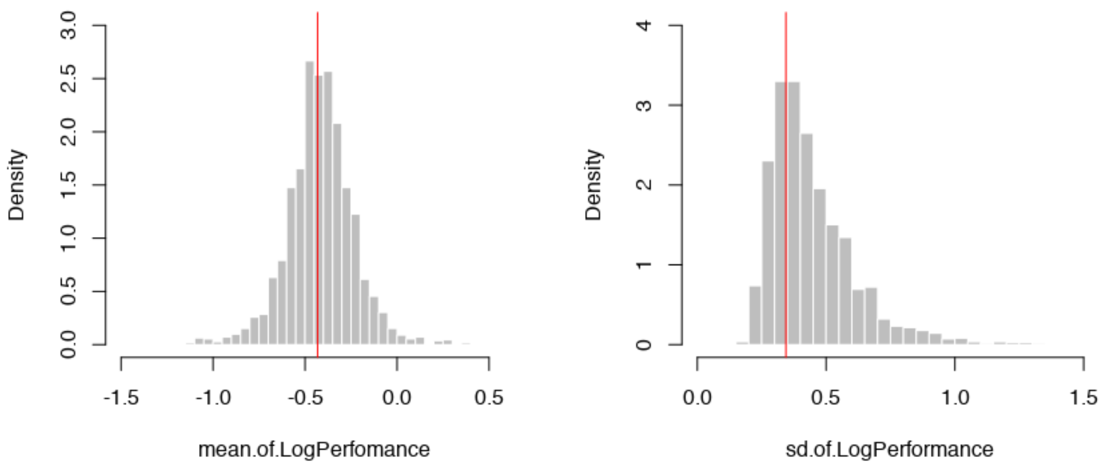


Figure 7.12
Histogram of samples from posterior distributions of hyperparmeters

7.5 Prediction Results

Table 7.8a lists the predicted mean anomalies for each spacecraft in the 5 fields based on the pseudo-space generated with input from all the respondents, while Table 7.8b shows the predictions using the ordination results from the sub-group. These results are from the Gaussian approximations of the Poisson distribution of anomalies, hence the fractional values instead of discrete numbers as would be expected from a Poisson. For sufficiently large values of the mean, the normal distribution is a good approximation of the Poisson, with a variance equal to the mean.

Comparison of the results in both tables shows a general increase in prediction accuracy when using expert opinion from the sub-group particularly for the Spacecraft E prediction.

The posterior Poisson distributions of the predicted anomalies for each spacecraft is shown in Figure 7.14. The blue line is the predicted mean while the red line marks the actual recorded number of anomalies. Predictions for Spacecraft F is poor in both cases and merits further analysis. For Spacecraft E, the performance improves significantly switching from the total group opinion input to the sub-group.

Table 7.8
Summary of prediction results

(a) Results from entire group of experts

Spacecraft	Duration	Anomalies		% Error	% Accuracy
		Actual	meanPred		
E	3922	163.00	107.98	33.76	0.66
F	4088	224.00	168.26	24.88	0.75
G.A	1673	16.00	9.63	39.80	0.60
G.B	1673	14.00	9.63	31.20	0.69
H.A	3808	67.00	63.97	4.52	0.95
H.B	3118	31.00	49.85	-60.80	0.39
I	5592	125.00	468.16	-274.52	-1.75

(b) Results from sub-group of experts

Spacecraft	**Duration**	**Anomalies**		**% Error**	**% Accuracy**
		Actual	**meanPred**		
E	3922	163.00	167.45	-2.73	0.97
F	4088	224.00	158.41	29.28	0.71
G.A	1673	16.00	11.94	25.39	0.75
G.B	1673	14.00	11.94	14.73	0.85
H.A	3808	67.00	65.07	2.89	0.97
H.B	3118	31.00	49.11	-58.43	0.42
I	5592	125.00	125.27	-0.22	1.00

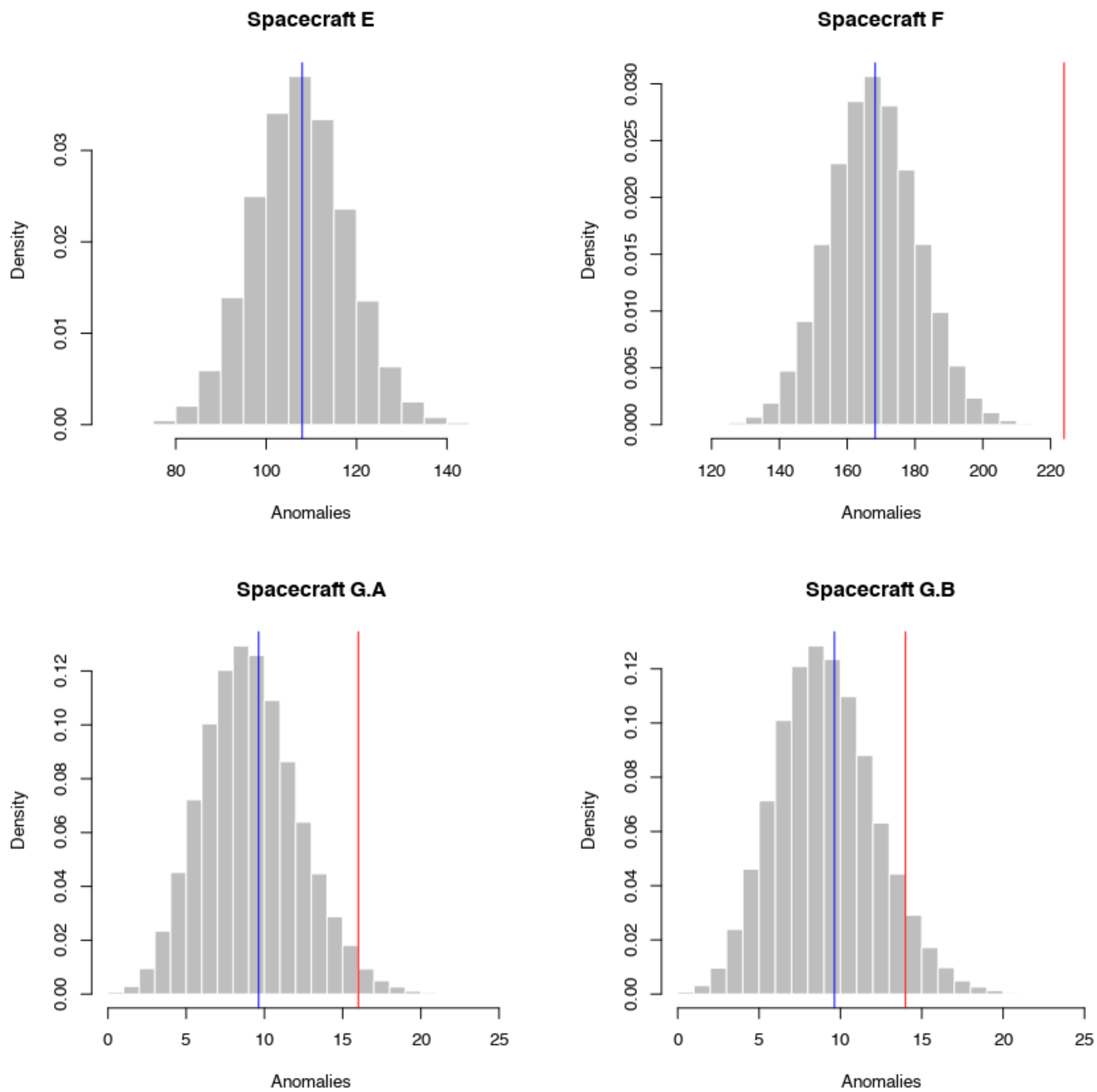


Figure 7.13

Posterior distributions of expected anomalies using all opinion data

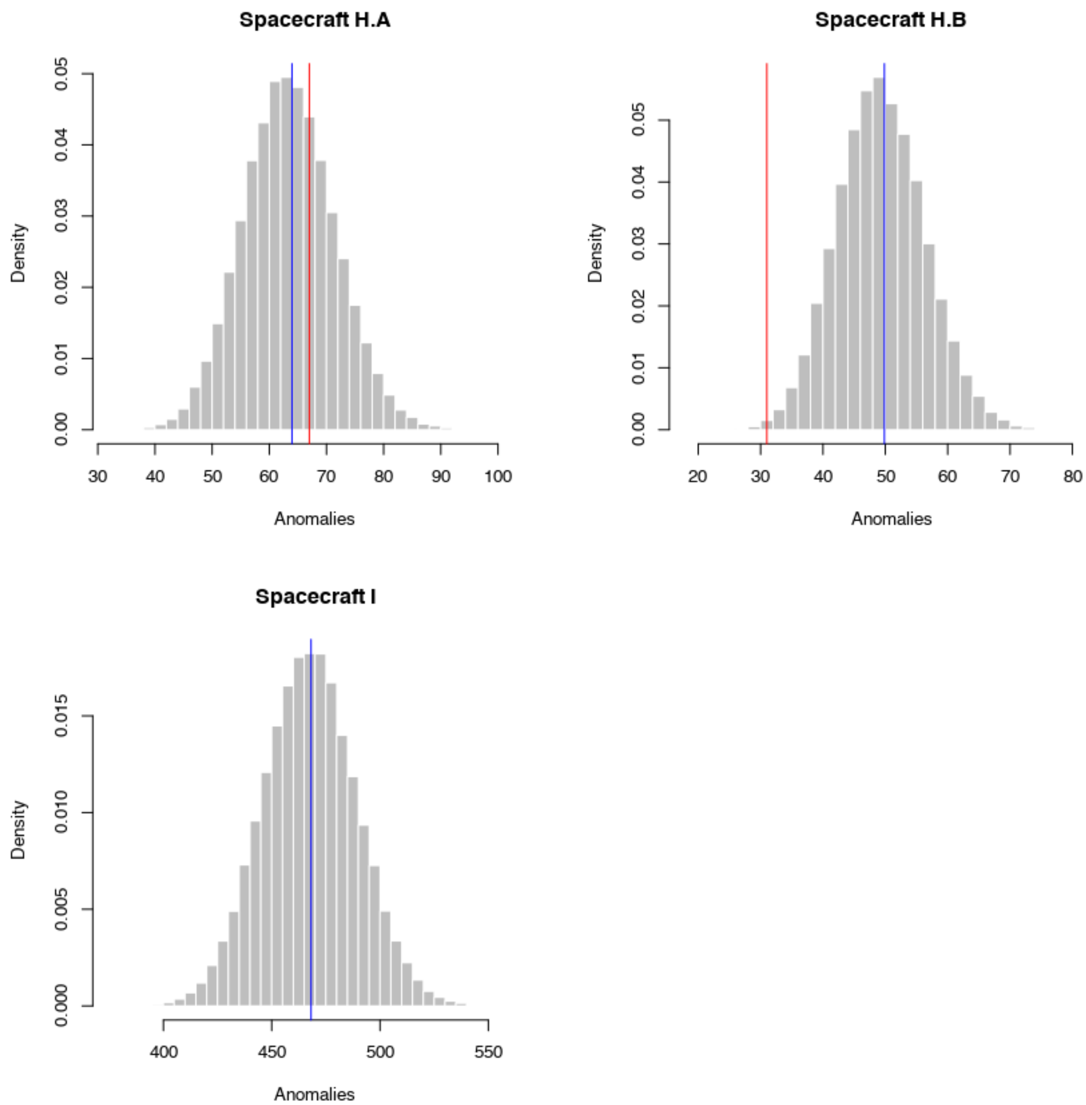


Figure 7.14

Posterior distributions of expected anomalies using all opinion data

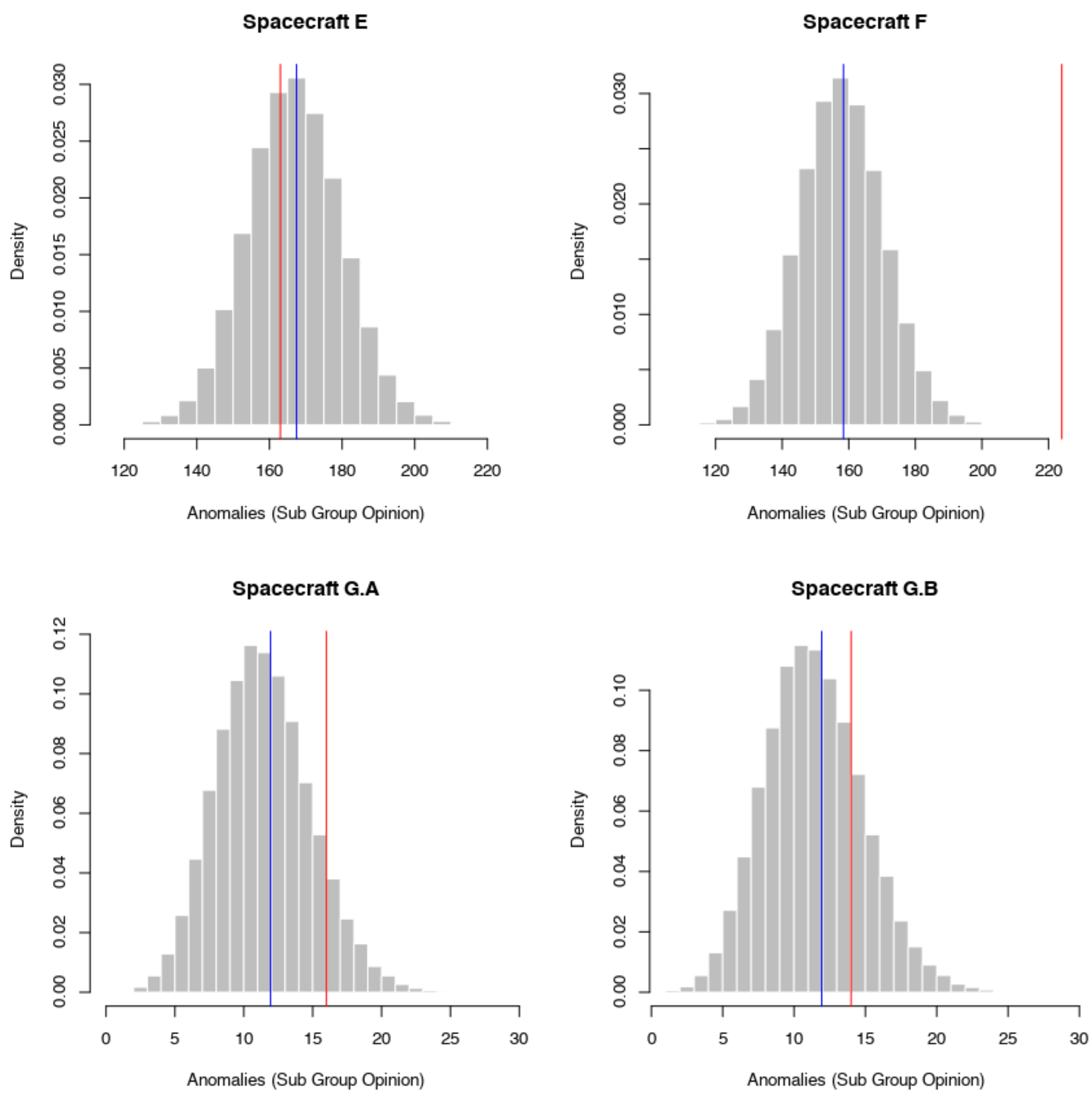


Figure 7.15

Posterior distributions of expected anomalies using SubGroup opinion data

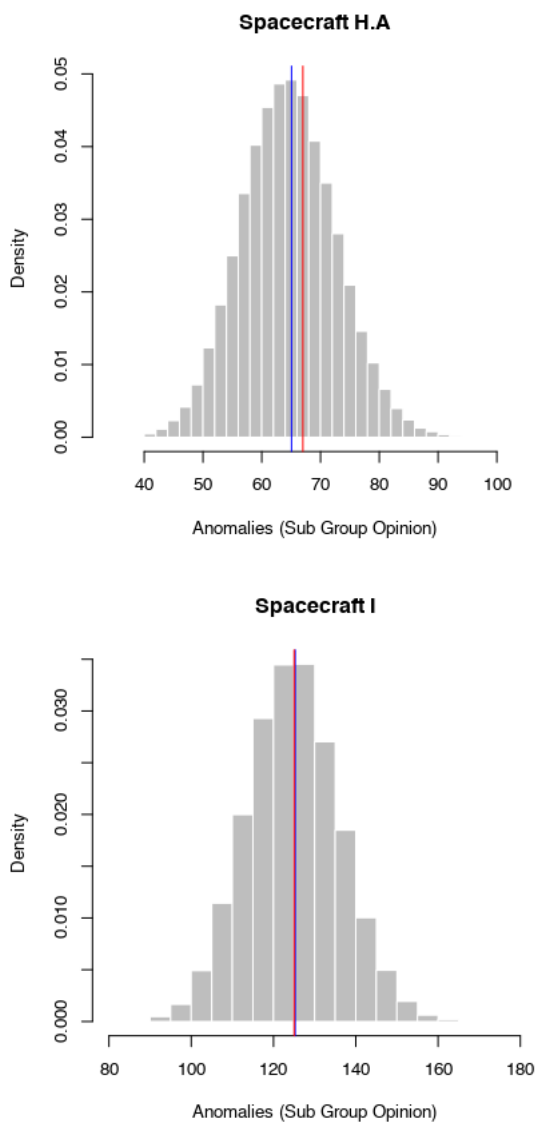


Figure 7.16

Posterior distributions of expected anomalies using SubGroup opinion data

7.6 Comparison with Point-Estimate of Population Mean

To compare the performance of our methodology against a simple estimate of the expectation of anomalies we treat the family of spacecraft as a homogeneous population and calculate an average. For a total of 640 anomalies from seven spacecraft, the population average is 91 anomalies. Comparing this expected value with the actual anomalies recorded for each spacecraft in Table 7.8 reveals how poorly such a point-estimate performs. Our methodology, on the other hand, estimates the individual mean anomalies with a much higher degree of accuracy for most of the spacecraft.

7.7 Alternative Trend Models

In reliability engineering, failure is typically regarded as an inevitable function of time; the longer a system operates, the higher the chances of it failing. This same trend is seen in the results of our model where the number of anomalies clearly increases as a function of mission duration. Given the anomaly and duration data in Table 7.8, we investigate other possible regressions, using the mission duration as the only explanatory variable in order to compare against our results.

7.7.1 Linear Trend

We first assume a linear trend fit to the data, as shown in Figure 7.17 and determine point estimates for the trend line parameters. Similar to the unique prediction fields for the pseudo-spatial analysis, the paramaters for the linear fit must be determined for each variation of the data set. That is, we must withhold the prediction point from the data set, estimate the trend line parameters, and then estimate and compare against the recorded number of anomalies. The Microsoft Excel© workbook for this is attached as Appendix F.

The results of the linear trend of anomalies as a function of mission duration is provided in Table 7.9. We also include the %*Error* associated with the prediction to enable comparison with the results from our pseudo-spatial trend.

7.7.2 Exponential Trend

We performed an exponential regression using the anomaly and duration data to round out the comparison against the performance of the pseudo-spatial model results. Similar to the linear trend, the coefficient and exponent parameters of the exponential fit lines are determined as point estimates for each of the five variations of the field. We provide the Microsoft Excel © worksheet

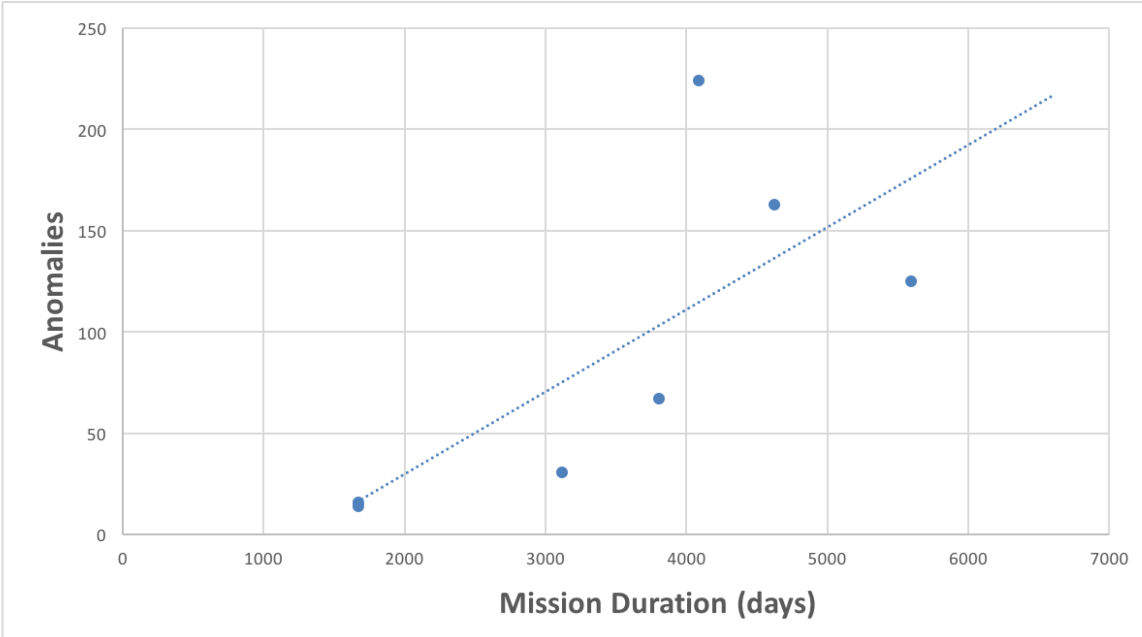


Figure 7.17

Linear fit to spacecraft anomaly data

Table 7.9

Anomaly prediction using a linear trend

Spacecraft	Duration	Anomalies		LP Accuracy	%Error
		Actual	Linear Prediction		
E	3922	163	101.92	0.63	37.47
F	4088	224	95.27	0.43	57.47
G.A	1673	16	47.44	-0.97	-196.50
G.B	1673	14	47.44	-1.39	-238.86
H.A	3808	67	114.91	0.28	-71.51
H.B	3118	31	88.25	-0.85	-184.68
I	5592	125	244.77	0.04	-95.82

for this estimation as Appendix F.

Figure 7.10 below is the exponential curve fit to the entire data set. Again, for each prediction, the target location is witheld from the data set prior to estimation and used only to assess the percentage error associated with the prediction.

Table 7.10 provides the set of predictions based on the data-specific exponential fit parameters.

7.8 Discussion

Bearing in mind that within the expert elicitation ground-rules, we relaxed the fidelity of the evaluation by reducing the dimensions of comparison from 42 to 8, the predictions for each spacecraft still falls well-within the one-order of magnitude target of the framework.

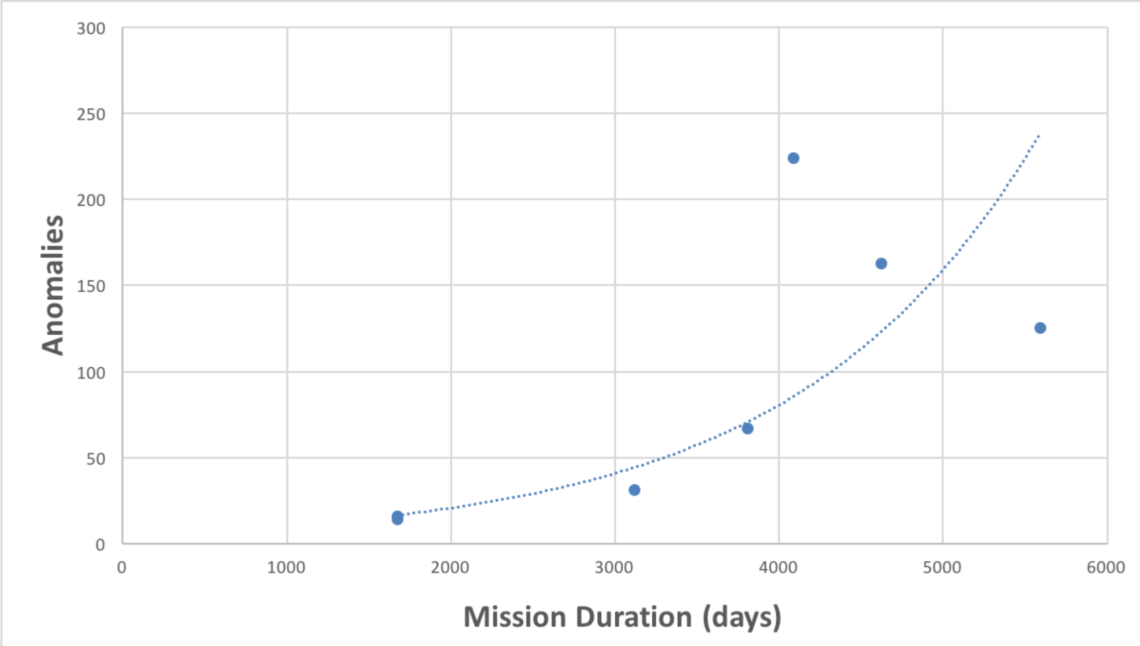


Figure 7.18

Exponential curve fit to spacecraft anomaly data

Table 7.10

Anomaly prediction using a exponential trend

Spacecraft	Duration	Anomalies		EP Accuracy	%Error Exp
		Actual	Exp Pred		
E	3922	163	71.49	0.44	56.14
F	4088	224	75.28	0.34	66.39
G.A	1673	16	32.74	0.05	-104.63
G.B	1673	14	32.74	0.34	-133.86
H.A	3808	67	84.25	0.74	-25.75
H.B	3118	31	52.68	0.30	-69.94
I	5592	125	617.44	2.94	-393.95

From the model predictions using the sub-group data, we observe an improvement in the %*Accuracy* and a reduction in the %*Error* of the mean predicted anomalies. This finding indicates that the comparison process should be vetted to ensure expertise is applicable across all dimensions of comparison. The proposed methodology is fundamentally driven by the goodness or accuracy of the expert input as evidenced by the sensitivity of the ordination solution to the dissimilarity matrix. By limiting the input to experts with broader-range expertise, enough to cover all pertinent attributes, we have improved the accuracy of the results. This suggests a potential trade between the degree of expertise sought and the desired depth of the comparison, but with the caveat that the comparison be limited to the specific attribute in question. The hierarchical taxonomy provides a structure to ensure that even such targeted comparison output can be upwardly incorporated in the overall assessment by virtue of its representation of the system.

The results from predictions suggest that there is indeed an underlying symbiance between the psychological space that subjectively creates proximity measures and applicability of hard data. We are extremely encouraged by these findings.

Comparing the performance results of the pseudo-spatial model with the linear trend and exponential trend, shown in Table 7.11[5], it is apparent the pseudo-spatial model, particularly the sub-group model, performs better in the five cases tested. By accounting for the pseudo-spatial correlation determined based on subjective opinion, our model improves on more traditional regression methods. While these regression approaches could conceivably include more explanatory variables to improve accuracy, the pseudo-spatial process is unique in that it provides a methodical approach to encoding the amorphous concept of subjective measures of system proximity. Additionally, we recognize the shortcomings of conducting point-estimation of the linear and exponential trend parameters; a fuller picture of the comparison may be obtained via Bayesian regression such that the attendant parameter uncertainty is also characterized. We defer these activities since our interest is in a quick comparison of expected performance.

Table 7.11

%Error Comparison; lower absolute values indicate smaller error

Spacecraft	% Error PSM	% Error PSM-SubGroup	%Error (LP)	%Error Exp
E	33.76	-2.73	37.47	108.09
F	24.88	29.28	57.47	-17.66
G.A	39.80	25.39	-196.5	36.21
G.B	31.20	14.73	-238.86	52.78
H.A	4.52	2.89	-71.51	36.19
H.B	-60.80	-58.43	-184.68	3.91
I	-274.52	-0.22	-95.82	99.92

An anecdotal detour; initial predictions for Spacecraft I estimated an average count of anomalies of 156 over its mission duration, however the actual number of anomalies recorded over the same duration in the database was eight. On discussing this discrepancy with the mission developer's chief engineer, who also was the mission systems engineer for Spacecraft I, we discovered that within the reporting system, a total of 117 anomalies and failures had been nested within other reports, effectively masking them from initial review of the data. The spacecraft maintains an orbit in which it is periodically exposed to increased harsh environments and as result certain anomalies reoccur. With the adjustment of the anomaly count and based on the associated error with this prediction has been significantly reduced.

[5] PSM: Pseudo-spatial model, EP: Exponential Predictor, LP: Linear Predictor

8 Limitations and Future Work

A number of elements impacted the course of this research. Of significance was the choice to aggregate the primary input to the spatial ordination given the eventual sensitivity of the results to its value. This is rather contrary to the Bayesian paradigm where every attempt is made to use all relevant evidence. The distance measure itself ought to reflect the variability in opinion such that a distribution of ordination solutions can be obtained. While Bayesian ordination is gaining ground, combining its results with the spatial modeling produces another challenge, hence the choice to adopt a singular representation of the pseudo-space.

In using the SPDE via Integrated Nested Laplace Approximations, we traded computing efficiency for a more intuituve handling of the Bayesian process. Although the transformation of the correlation function parameters to the SPDE parameters is explicitly defined in the literature and addressed in this discourse, the consequence of multiple transformations necessitate the assumption of independence to simplify the process. This limitation, though, is purely an implementation issue that can be addressed with a deeper study into the relationship between the dissimilarity matrix and the Matern parameters of the pseudo-spatial field.

From a qualitative perspective, the choice to condense the dimensions of comparison was made to ensure adequate number of participants in the study. From the results of the sub-group analysis, we believe that a more comprehensive assessment would have provided even better results. But we adusted our expectations based on the fact that the method must work with minimal input. Eliciting evaluations on 36 pairs of systems across 42 attributes may have led to no affirmative responses to our elicitation invitation based on the daunting scope of performing 1512 pairwise comparisons. Developing a process for evaluation of the importance of attributes to aid in reducing the pairwise comparisons, even with stated pertinence to the context of comparison, would be of benefit in consolidating attributes.

The methodology presented in this research provides a tool for assessing the relevance of information across variants of any engineered system such that probabilistic assessment of any conceptual variant can be conducted by quantifying similarity. The framework however can serve as a springboard for integrating other analysis elements that would fill some of the stated limitations. Hence motivated by the foregoing revelations of issues encountered in this study, we present a few possibilities for expanding on and improving the resultant methodology.

8.1 Bayesian Ordination

Key to the success of the method is the expert judgment with which to formulate the pseudo-spatial configuration of the family of systems. Our assessment of the sensitivity of the model results indicate that variability of the pseudo-space resulting from the opinion data is a significant driver of the results. In seeking to capture the uncertainty due to the possible variations in the pseudo-spatial configuration, we turn Bayesian inference once again.

By treating the pairwise elements of the dissimilarity matrix that is fed into the ordination process as random variables, and implementing a Bayesian multidimensional scaling, one can generate a probability density of possible ordination plots. Such a probability density will contain the uncertainty propagated through the random-valued pairwise dissimilarity measures.

8.2 Integration of Metrically and Subjectively Derived Psuedo-spatial Configurations

Another aspect of the methodology is the integration of metric measures of similarity with subjectively derived distance measures. Metric attributes pertinent to the context of comparison can be directly compared to determine degree of similarity. This would yield dissimilarity matrices that can then be used in metric multidimensional scaling.

We have focused so far on using psychological measures of proximity to build a dissimilarity structure, an extension of the methodology would develop an integration scheme that would result in a pseudo-spatial configuration derived from both metric and nonmetric ordination.

8.3 Bayesian Importance Analysis of Comparison Attributes

We also envision incorporating a probabilistic treatment of the importance of attributes in inter-variant comparison. By ascribing degrees of importance to different attributes or attributes with respect to the context of comparison, one can potentially attenuate the impact of those attributes on the results, or altogether eliminate them from the assessment. The degree of importance could be handled as a random variable updated with evidence through a Bayesian process.

9 Conclusion

On the premise that there exists a single metric of proximity between all manner of things when contextually compared, we sought to develop a methodological and systematic approach for inference based on historical information convinced that such a metric would determine the appropriateness of the information. With the seminal work of Roger N. Shepard on the universal law of generalization in psychological sciences serving as a springboard for our investigation, we proposed that the concept of a psychological space is valid in assessing proximity of engineered systems.

In advancement of our research, we first ascribe the learning and adaptation elements attendant in the mapping of stimulus-response processes in behavioral sciences to the human design engineering context. The link being that a designer implicitly imparts, albeit indirectly, learning on exisitng engineered systems such that the systems gradually evovle over time. These existing, or conceptual, systems represent comparable stimuli that can be mapped to demonstrated behavior or responses such that, as system designs become more similar, the chances of generalizing behavior from one to another increases. With the foregoing, our task crystallized into the quantification of proximity between stimuli as a singular metric in psychological space, and utilizing the invariant exponential law of spatial relatedness to aid the inference or response mapping endeavor.

This thesis, has furthered exisiting work in the area of generalization and behavior mapping by demonstrating that inferential analysis on engineered systems, when informed largely by subjective but expert, human judgement, can lead to valid quantitative results. Along the way, we have developed an assessment framework for using historical information in probabilistic analysis in which demonstrated behavior informs future behavior. The critical input to the framework are the rank-ordered proximity values which enable the construction of a geometric representation of psychological space, which subsequently bridges the gap between the physical and the conceptual.

Combined with potential areas of future research discussed previously, we foresee the expansion of our methodology in several applications of risk analysis in early design. Numerous instances abound of the use of subjective input in characterizing the applicability of information. Often times, these devolve into making educated cases. While not necessarily a poor estimation strategy, the lack of mathematical rigor in ascribing the "guess", coupled with potentially high consequences of ill-informed guesses, entrench the demure attitude of large scale development projects towards such opinion-based methods of risk assessment.

By demonstrating the viability of our methodology, we have established a traceable and structured process for assessing risk with very limited design information, but with adequate design know-how. To introduce a mathematical formalism to the guessing process, we deconstructed the problem into elementary parts, adopted proven but seemingly out of context methods for addressing each element, and finally, leveraged the elegance of Bayesian theory to re-integrate the elements into a functional framework.

In closing, it is our belief that through the use of pseudo-spatial models of comparable engineered systems, development projects faced with the heightened risk of conceptual design phase, coupled with the uncertainty of new technology, will be equipped to, non-comittally, and for a fraction of the cost of traditional feasibility studies, obtain a high-level understanding of risk.

A Elicitation Invitation

Good afternoon,

I am writing to request your participation in a brief survey intended to elicit your opinion on the degree of similarity between a selection of **redacted** designed and built spacecraft. Your years of experience at **redacted** and the varying degrees to which you have participated on spacecraft development projects will ensure the completeness of the survey data, hence my appeal.

Although I have received approval from **redacted** to conduct this survey, the effort is not tied to any **redacted** project. The data collected will be used to validate the results of my research in the use of psychological/intuitive measures of proximity in characterizing the spatial relationship of a family of complex systems. The effort is not a referendum on the soundness of **redacted**-designed spacecraft and all references to sponsors, projects, and survey participants will be redacted in my dissertation and any other publications.

The survey question, your opinion of the similarity on a scale of 1 to 9 between 36 pairs of spacecraft, will take approximately 45 minutes to complete. Please email me confirmation of your willingness to participate and I will provide you with the survey material.

I sincerely hope that you will agree to participate and I look forward to sharing the details and results of my research with you. Thank you to those that have already committed their time to this effort.

Sincerely, Obi Ndu

B Elicitation Results

X	Pair.ID	Expert.1	Expert.2	Expert.3	Expert.15	Expert.16	Expert.17
Pairs	NA	x	x	x	x	x	x
A-B	1	7		6	4	4	2
A-C	2	8		5	4	7	3
A-D	3	8		5	7	4	2
A-E	4	8		2	3	5	2
A-F	5	6		2	3	4	2
A-G	6	7		6	6	5	1
A-H	7	7		8	7	5	1
A-I	8	7		5	3	5	2
B-C	9	6		5	2	5	8
B-D	10	6		3	4	3	2
B-E	11	6		5	4	5	3
B-F	12	7		1	3	4	2
B-G	13	8		6	4	4	1
B-H	14	7		5	4	4	1
B-I	15	7		8	3	4	2
C-D	16	8	6	7	7	4	3
C-E	17	8	4	7	5	5	3
C-F	18	6	3	6	5	5	3
C-G	19	7	3	3	4	4	2
C-H	20	7	3	6	5	5	2
C-I	21	7	4	7	6	5	4
D-E	22	8	3	7	8	5	3
D-F	23	6	3	8	6	6	7
D-G	24	7	3	3	4	4	2
D-H	25	7	3	6	5	4	2

Expert.20	Expert.25	Expert.28	Expert.30	Expert.31	Expert.32	Expert.33
x	NA	x	x	x	x	x
5	4.33	4	5	1		7
7	4.50	5	5	1		7
2	4.00	3	5	2		6
1	2.67	2	4	1		4
1	3.50	1	4	1		5
4	3.67	1	4	6		3
6	3.83	2	5	1		5
4	3.83	3	4	1		6
	4.83	4	7	3		8
	4.33	2	6	2.5		7
	3.17	2	5	3		5
	3.33	1	5	2		6
	3.83	1	4	1		4
	4.00	1	5	3		6
	4.17	3	5	5		7
6	4.83	7	5	2.5		7
4	3.67	6	5	4	3	5
3	4.17	3	5	3	1	6
2	4.00	2	5	1	2	4
3	4.17	2	6	3	3	6
4	4.50	4	5	3	2	7
4	3.17	7	6	2.5		6
3	4.17	4	7	7		8
3	4.00	3	6	2		5
5	4.50	3	7	2.5		7

C Algorithms

C.1 Spacecraft Ordination via SMACOF

The following redacted R code, was developed using the SMACOF package, [@DeLeeuw2009]

```
##----SpacecraftSMACOFchunk----
graphics.off()
rm(list=ls(all=TRUE))
rm(list=ls())

# library (vegan)
library(MASS)
library(xlsx)
library(smacof)
library(XLConnect)
Spacecraft.data <- read.xlsx ('R_INLA_Models/Final_ExpertData.xlsx', 8,
row.names = 1, head = T)
names (Spacecraft.data) <- rownames (Spacecraft.data)

ndim = 7
Spacecraft.Results = vector("list", ndim)
for(i in 1:ndim){
  Spacecraft.Results[[i]] = smacofSym(Spacecraft.data, ndim = i, type = c( "ordinal"),
                  weightmat = NULL, init = "torgerson", ties = "primary", verbose = FALSE,
                 relax = FALSE, modulus = 5, itmax = 1000, eps = 1e-06,
                    spline.degree = 2, spline.intKnots = 2)
}


summary(Spacecraft.Results[[3]])
```

```
ab = Spacecraft.Results[[3]]$stress
ab4 = Spacecraft.Results[[4]]$stress
xD1 = t(t(Spacecraft.Results[[3]]$conf[,1]))
yD2 = t(t(Spacecraft.Results[[3]]$conf[,2]))
zD3 = t(t(Spacecraft.Results[[3]]$conf[,3]))

SpacecraftStress = sapply(Spacecraft.Results,
function(Spacecraft.Results)Spacecraft.Results$stress)
x11(type = "cairo", height = 4, width = 6)
plot(1:ndim, SpacecraftStress[1:ndim],
     xlab = "Dimensions", ylab = "Kruskal Stress",
     type = "o", pch = 21, cex = 1, bg = "red", lwd = 1, font = 2)
abline(h = ab, lwd = 1, lty = 2, col = "red")
```

C.2 Anomaly Prediction via R-INLA

The following redacted code was developed using methods descibed in [blangiardo2015]

```
graphics.off()
rm(list=ls(all=TRUE))
rm(list=ls())
library(ggplot2)
library(INLA)
library(xlsx)
library(data.table)
library(mcmcplots)
library(Matrix)
library (vegan)
library(MASS)
library(xlsx)
```

```
library(calibrate)
library(smacof)
# library(fields)
library(GoFKernel)
library(geoR)
library(gridExtra)
library(png)
library(grid)

#+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
# Data Preparation
# Import coordinates from the the SMACOF Spatial Configuration
# The coordinates have been shifted but the configuration is unaffected

# Select which data set
# SpacecraftFile = as.data.frame(read.xlsx("SpacecraftCoords copyAugmented.xlsx", 2))
SpacecraftFile = as.data.frame(read.xlsx("SpacecraftCoords copyCorrected_.xlsx", 2))




# SpacecraftFile = as.data.frame(read.xlsx("SpacecraftCoords.xlsx", 2))
# SpacecraftFile = as.data.frame(read.xlsx("SpacecraftCoords2A.xlsx", 1))

est.coords = as.matrix(na.omit(cbind(SpacecraftFile$xShiftedEst,
   SpacecraftFile$yShiftedEst)))
est.data = as.matrix(na.omit(SpacecraftFile$AnomalyEst))
est.rate.data = as.matrix(na.omit(SpacecraftFile$RateEst))
est.duration = as.matrix(na.omit(SpacecraftFile$DurationEst))
est.z = as.matrix(na.omit(SpacecraftFile$AltitudeEst))

val.coords = as.matrix(na.omit(cbind(SpacecraftFile$xShiftedVal,
    SpacecraftFile$yShiftedVal)))
val.coords.2 = val.coords[-5,]
```

```
val.data = as.matrix(na.omit(SpacecraftFile$AnomalyVal))
val.z = as.matrix(na.omit(SpacecraftFile$AltitudeVal))
val.duration = as.matrix(na.omit(SpacecraftFile$DurationVal))

domainX = as.matrix(na.omit(as.data.frame(SpacecraftFile$domainX)))
domainY = as.matrix(na.omit(as.data.frame(SpacecraftFile$domainY)))
#++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++


# To create domain, look at plot of coords in excel
domain = matrix(cbind(domainX, domainY), ncol = 2)

mesh = inla.mesh.2d(loc.domain = domain,
                    max.edge = c(0.04, 0.2),
                    cutoff = 0.05,
                    offset = c(0.1, 0.1))

x11(type = "cairo")
plot(mesh, main ="")
points(est.coords, pch = 21, bg = 1, col= "white", cex = 1.8)
points(val.coords, pch = 21, bg = "red", col= "black", cex = 1.8)
points(val.coords.2, pch = 21, bg = "yellow", col= "black", cex = 1.8)

#+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
# Select which plot to save, be sure to match with data set

# savePlot(filename = paste("SpacecraftTriangulation.png", sep = "."),
type = c("png"), device = dev.cur())

savePlot(filename = paste("F1_Mesh_Spacecraft.png", sep = "."),
type = c("png"), device = dev.cur())

#++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
vertices = inla.spde2.matern(mesh, alpha = 2)$n.spde #+++++++> returns the
number of vertices for a mesh
#+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Spacecraft.est = inla.spde.make.A(mesh = mesh, loc = est.coords)
Spacecraft.val = inla.spde.make.A(mesh = mesh, loc = val.coords)

dim(Spacecraft.est)
table(apply(Spacecraft.est, 1, nnzero))
table(apply(Spacecraft.est, 2, sum) > 0)

#++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
#+++++++ Estimation of model parameters +++++++++++++++++++++++++++++++

# 1. create a Matern SPDE object
range0 = 5
sigma0 = 1
kappa0 = sqrt(8)/range0
tau0 = 1/(sqrt(4*pi)*kappa0*sigma0)
Spacecraft.spde = inla.spde2.matern(mesh = mesh, alpha = 2,
                 B.tau = matrix(c(log(tau0),-1,+1), nrow = 1, ncol = 3),
                B.kappa = matrix(c(log(kappa0),0,-1), nrow = 1, ncol = 3),
                # B.tau = matrix(c(0,1,0), nrow = 1, ncol = 3),
                       # B.kappa = matrix(c(0,0,1), nrow = 1, ncol = 3),
                        # prior.tau = c(.05),
                             # prior.kappa = c(7))
                               theta.prior.mean = c(0,0),
                               theta.prior.prec = c(1,1))

# 8. Alternative estmation of parameters using STACK
s.index = inla.spde.make.index(name = "Spacecraft.spatial.field",
```

```
        n.spde = Spacecraft.spde$n.spde)

names(s.index)
s.index$Spacecraft.spatial.field = seq(1, Spacecraft.spde$n.spde)
s.index$Spacecraft.spatial.field.group = rep(1, Spacecraft.spde$n.spde)
s.index$Spacecraft.spatial.field.repl = rep(1, Spacecraft.spde$n.spde)

Spacecraft.stack.est = inla.stack(data = list(y = est.data),
        A = list(Spacecraft.est, 1), #increase effects list if you add more
                 effects = list( c(s.index, list(intercept = 1)),
                       list(Duration = est.duration)),
                                  # list(Altitude = est.z)),
                             tag = "est") # Estimation

Spacecraft.stack.val = inla.stack(data = list(y = NA), #predict and compare
                             A = list(Spacecraft.val, 1),
                             effects = list( c(s.index, list(intercept = 1)),
                           list(Duration = val.duration)),
                          list(Altitude = val.z)),
                             tag = "val") # Estimation

Spacecraft.join.stack = inla.stack(Spacecraft.stack.est, Spacecraft.stack.val)

formula = y~ -1 + intercept + Duration +f(Spacecraft.spatial.field, model = Spacecraft.spde)

Spacecraft.output = inla(formula,
           data = inla.stack.data(Spacecraft.join.stack, spde= Spacecraft.spde),
             family = "poisson", E = 1,
              control.predictor =list( A=inla.stack.A(Spacecraft.join.stack), compute=TRUE),
          control.compute = list(cpo = TRUE, dic=TRUE))
# Prediction at the validation locations
index.val = inla.stack.index(Spacecraft.join.stack, "val")$data
```

```
post.mean.val =
  round(Spacecraft.output$summary.linear.predictor[index.val, "mean"], 4)
post.sd.val =
  round(Spacecraft.output$summary.linear.predictor[index.val, "sd"], 4)

# Prediction at the estimation locations
index.est = inla.stack.index(Spacecraft.join.stack, "est")$data

post.mean.est =
  round(Spacecraft.output$summary.linear.predictor[index.est, "mean"], 4)
post.sd.est =
  round(Spacecraft.output$summary.linear.predictor[index.est, "sd"], 3)
summary(Spacecraft.output)
exp(post.mean.val)
exp(post.mean.est)

output.test = inla.spde2.result(inla = Spacecraft.output,
                                name = "Spacecraft.spatial.field",
                                spde = Spacecraft.spde, do.transf = TRUE)

summary(output.test$marginals.kappa$kappa.1)
inla.emarginal(function(x) x, output.test$marginals.kappa[[1]])



meanPred = EstimatedAnomalies_1
Actual = 163
Model.Percent.Accuracy = 1-abs(meanPred - Actual)/Actual
Spacecraft_E_Results = data.frame(meanPred, Stand.dev_1,
Model.Percent.Accuracy, row.names = "Spacecraft E")
Spacecraft_E_Results
write.xlsx(Spacecraft_E_Results, "SpacecraftResults_corrected.xlsx",
  "Spacecraft_E_Results", append = TRUE)
```

C.3 Model Prediction Perfomance Updating via RJAGS

The following code was developed based on material in John Kruschke's *Doing Bayesian Data Analysis* ***???*** *Edition*, [57].

```
#1. THE DATA
ModelPerformance.Data = read.xlsx("SpacecraftResults.xlsx", 6)
ModelPerformance.Data.SubGroup = as.vector(na.omit( ModelPerformance.Data$X..Accuracy1))
Perf = ModelPerformance.Data.SubGroup
LogPerf = log(ModelPerformance.Data.SubGroup)
Ndata = length(LogPerf)
trueLogM = mean(LogPerf)
trueLogSD = sd(LogPerf)

y = Perf
N = length(y)
meanOfLogY = mean(log(y))
sdOfLogY = sd(log(y))



ModelPerformance.Data.List = list(
  y = y ,
  N = N ,
  meanOfLogY = meanOfLogY ,
  sdOfLogY = sdOfLogY
)

#X. THE MODEL
ModelPerformance = "

 model {
    for( i in 1 : N ) {
      y[i] ~ dlnorm( muOfLogY , 1/sigmaOfLogY^2 )
    }
```

```
  sigmaOfLogY ~ dunif( 0.001*sdOfLogY , 1000*sdOfLogY )
  muOfLogY ~ dnorm( meanOfLogY , 1/(10*sdOfLogY)^2 ) # updated 8/16/2017
  muOfY <- exp(muOfLogY+sigmaOfLogY^2/2)
  modeOfY <- exp(muOfLogY-sigmaOfLogY^2)
  sigmaOfY <- sqrt(exp(2*muOfLogY+sigmaOfLogY^2)*(exp(sigmaOfLogY^2)-1))

} # Close model
"
writeLines(ModelPerformance, con="ModelPerformance.txt")

library('rjags')
parameters =  c("muOfLogY" , "sigmaOfLogY" , "muOfY" , "modeOfY" , "sigmaOfY" )


# Create, initialize, and adapt the model:
jagsModel = jags.model( "ModelPerformance.txt" , data=ModelPerformance.Data.List ,
                        n.chains=nChains , n.adapt=adaptSteps)# inits = initsList )
# Burn-in:
cat( "Burning in the MCMC chain...\n" )
update( jagsModel , n.iter=burnInSteps )
# The saved MCMC chain:
cat( "Sampling final MCMC chain...\n" )
mcmcCoda = coda.samples( jagsModel , variable.names=parameters ,
                            n.iter=15000 , thin=thinSteps )


summary(mcmcCoda)
```

D Anomaly Data

Anomaly Data for Spacecraft E are attached as a spearate spreadsheet.

E Space Mission Taxonomy

Generic hierarchical taxanomy of spacecraft

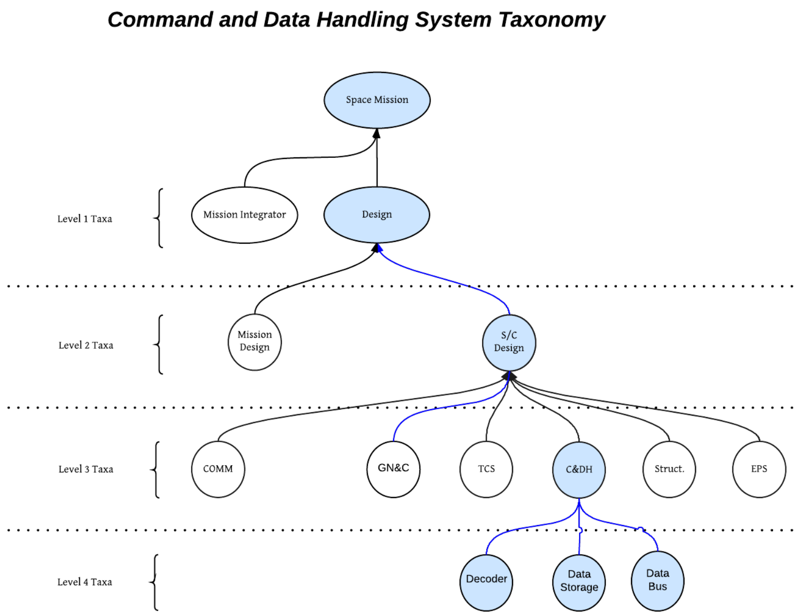


Figure E.1

Command and Data Handling Taxonomy

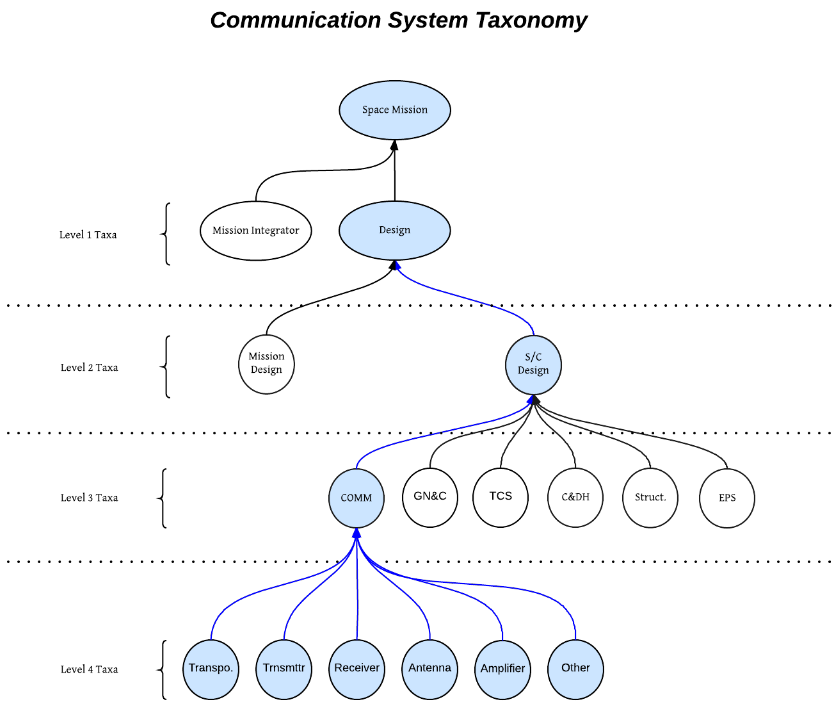


Figure E.2

Communications Subsystem Taxonomy

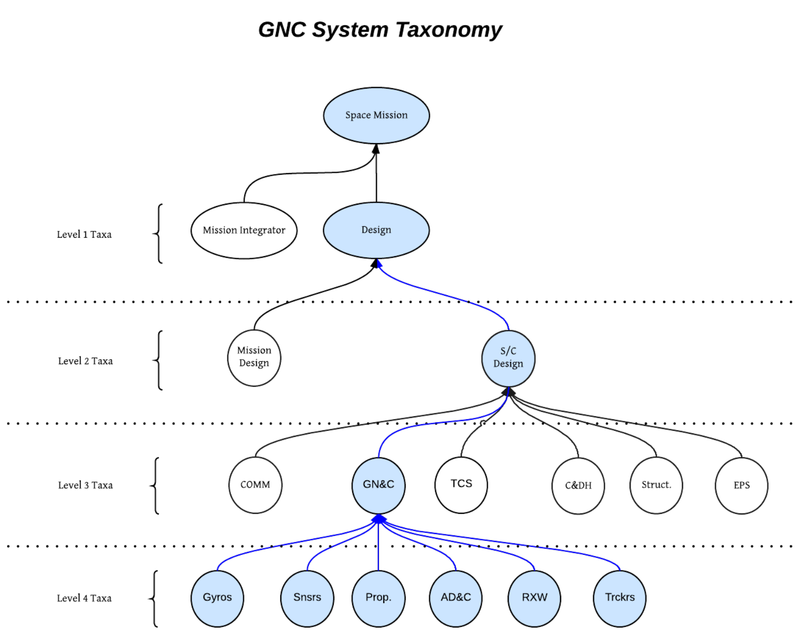


Figure E.3

GNC Subsystem Taxonomy

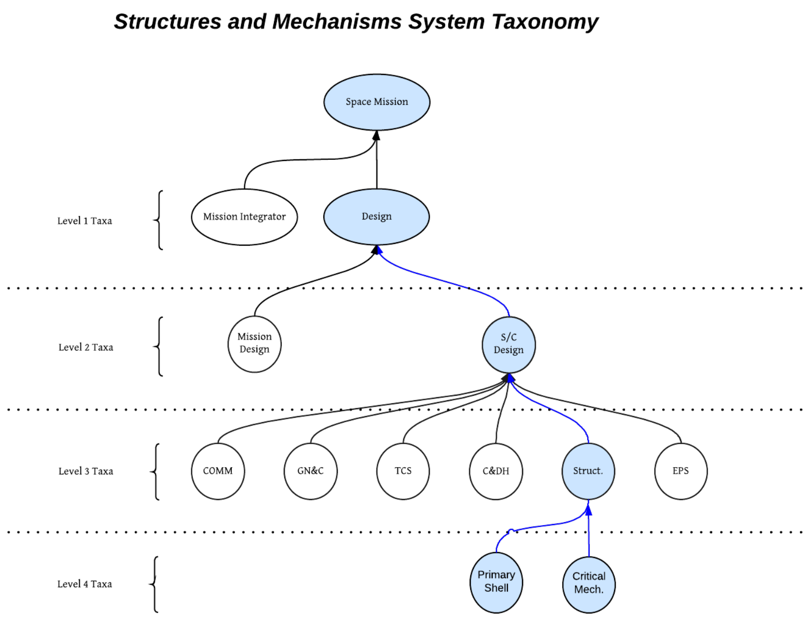


Figure E.4

Structures and Mechanisms Taxonomy

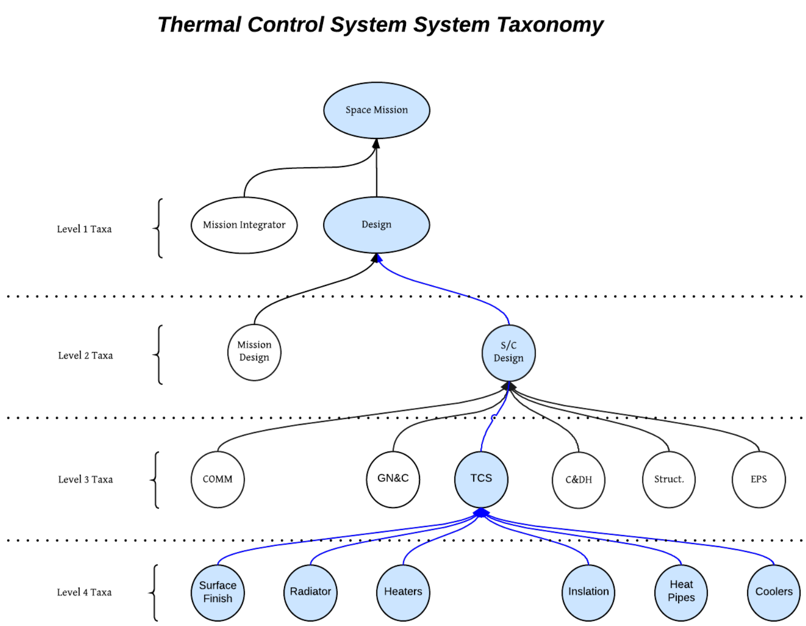


Figure E.5

Thermal Control Subsystem Taxonomy

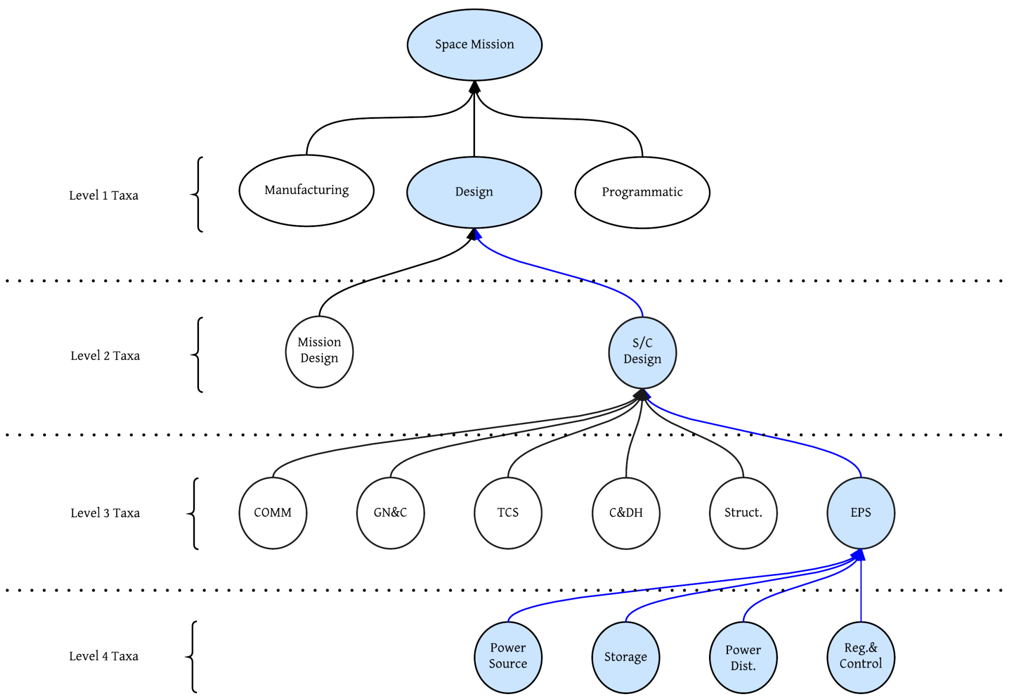


Figure E.6

Electrical Power Subsystem Taxonomy

Table E.1

Set of factors for spacecraft comparison

Full Set of Factors				
Goal	Level 1 Comparators	Level 2 Comparators	Level 3 Comparators	Level 4 Comparators
Spacecraft Similarity	Design	Mission Design	Orbit/Trajectory	Orbit/Trajectory
			Space Environment	Radiation MMOD Plasma Atomic Oxygen
		Spacecraft Design	Communication	Transponder Transmitter Receiver Antenna Amplifier
			GN&C	Gyros Sensors Propulsion AD&C RXW Trackers
			Env. Control	Radiator Heaters Insulation Heat Pipes Coolers
			C&DH	Processor Data Storage Data Bus
			Struc. & Mech.	Primary Shell Critical Mechanisms
			EPS	Source Storage Distribution Reg. & Control
	Integrator/Developer	PM&O	Risk Management	Risk Mangement Experience Risk Mangement Methods and Tools
			Leadership/Non Technical Factors	Project Management Staff Management Skills Financial Management
			Technical Management	Technical Management/Systems Engineering
		Manufacturing and Quality	Quality Assurance	Quality Assurance Methods Quality Assurance Tools
			Configuration Control	Configuration Control Methods Configuration Control Tools
			Process Control	Process Control Methods Process Control Tools

F Linear and Exponential Trend

Spreadsheets are attached.

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